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## Two conferences

Lars-Erik Persson

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## TWO CONFERENCES

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Série de deux conférences prononcées les 6 et 10 novembre 2015 (invitation sur proposition du Pr Pierre-Louis Lions). Ces conférences sont disponibles en vidéo sur le site internet du Collège de France : <https://www.college-de-france.fr/site/en-pierre-louis-lions/guestlecturer-2015-2016.htm>.

## HARDY TYPE INEQUALITIES: PREHISTORY, HISTORY AND CURRENT STATUS

The first weighted form of the continuous form of Hardy's inequality reads:

$$\int_0^{\infty} \left( \frac{1}{x} \int_0^x f(y) dy \right)^p x^{\alpha} dx \leq \left( \frac{p}{p-1-\alpha} \right)^p \int_0^{\infty} f^p(x) x^{\alpha} dx, \quad (1)$$

where  $f$  is a measurable and non-negative function on  $(0, \infty)$ ,  $p > 1$ ,  $\alpha < p - 1$ . First I present Hardy's original motivation from around 1915 and some historical facts until Hardy in 1925 finally proved the first version of (1) (with  $\alpha = 0$ ). After that I present the easiest proof of (1) I know, which was not discovered by Hardy himself. I continue by presenting some steps in the remarkable development of (1) to what today is called Hardy type inequalities, see e.g. the books<sup>1</sup> and the references therein. In particular, I present a choice of examples of interesting questions and results from each of the Chapters in the book<sup>2</sup> including the following:

1. Characterizations of inequalities of type (1) with general weights and various parameters.
2. The study of the same problem but with general kernel involved.
3. Higher order Hardy inequalities.
4. Fractional order Hardy inequalities.
5. The study of the same problem on the cone of monotone functions.
6. A remarkable relation to interpolation theory.

A number of open questions will be pointed out. In the final part of this lecture I will present some newer results, which can not be found in the cited books above and which in some cases contribute to give new light to the mentioned open questions. In particular, nowadays it is known that

1. A. Kufner et L.E. Persson, *Weighted Inequalities of Hardy Type*, NJ, World Scientific Publishing Co. Inc., River Edge, 2003 ; V. Kokilashvili, A. Meskhi et L.E. Persson, *Weighted Norm Inequalities for integral Transforms with Product Kernels*, New York, Nova Scientific Publishers, Inc., 2010.

2. A Kufner, L. Maligranda et L.E. Persson, *The Hardy Inequality. About its History and Some Related Results*, Pilsen, Vydavatelsky Publishing House, 2007.

- a. “all” powerweighted Hardy inequalities of the type (1) are more or less equivalent,
- b. the (Muckenhoupt-Bradley type) conditions found in 1. above can be replaced by infinite many alternative conditions (even four scales of conditions),
- c. sharp constants can now be found in many situations, which has not been known for long time,
- d. there exist new possibilities to develop the multidimensional theory, which was almost stopped after a result by Sawyer in 1985.

#### THE INTERPLAY BETWEEN CONVEXITY, INTERPOLATION AND INEQUALITIES

Different types of inequalities are very important in various areas of mathematics and its applications. Today the knowledge about inequalities has been developed to be an independent area with many papers, Journals, conferences and books (29 of them are listed in Appendix I).

Interpolation theory was partly motivated by developing a theory for better understanding of some inequalities (e.g. Hausdorff-Young’s inequality) and has now been developed to a fairly independent area of great interest also for several applications (see e.g. the eight books listed in Appendix I).

Convexity is one of the most fundamental concepts in analysis and various applications (see e.g. the eight books listed in Appendix I, in particular<sup>3</sup> [B] below).

The core of this lecture is to give a number of examples how the developments of these areas have supported each other and that it is still an intensive interplay between the continued developments.

I begin by pointing out the fact that several (more than 10) of the most common inequalities in analysis books are more or less simple consequences of the concept of convexity. After that I shortly describe some examples of the interplay between the developments of (complex and real) interpolation theory and the theory and understanding of other inequalities. The aim is to have a more unified approach to derive inequalities than in other inequality books, see<sup>4</sup>.

Just as one example how it is possible to develop these ideas further I present some generalizations of Carlson’s inequalities, which were important for the development of the real interpolation method (J.L. Lions, J. Peetre, and others). After that I present some further developments, which help to understand and further develop the Peetre  $\pm$  interpolation method and its applications (e.g. with this method we can even interpolate between certain Orlicz spaces), see also<sup>5</sup>.

A number of open questions are raised.

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3. C. Niculescu et L.E. Persson, *Convex Functions and their Applications - A Contemporary Approach*. Canad. Math. Series Books in Mathematics, Springer, 2006.

4. L.E. Persson et N. Samko, *Classical and New Inequalities via Convexity and Interpolation*. à paraître.

5. L. Larson, L. Maligranda, J. Pecaric et L.E. Persson, *Multiplicative Inequalities of Carlson Type and Interpolation*, World Scientific Publishing Co., 2006.