Proofs – are they objective?

Dag Prawitz

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will be considered. It will be argued that in her heavenly aspect she corresponds to the Gnostic and Christian Holy Spirit, and was considered in Assyria the feminine aspect of the supreme God.

M. Dag Prawitz
Professeur, université de Stockholm (Suède)

Proofs – are they objective?

The concept of proof may be given a first approximate explanation by saying that a proof is a chain of valid inferences from known truths such that at each inference step the conclusion is seen to follow from the premisses. A natural reaction to this explanation is to say that whether something “is seen to follow” may depend on a subject, and to ask whether this does not make proofs subjective in character.

We cannot simply skip the phrase “is seen to” in this explanation, if the validity of an inference is understood in the customary way in terms of necessary truth preservation or consequence. Doing so would give an explanation according to which every theorem of an axiomatic theory has a one-step proof, consisting of an inference that simply takes sufficiently many of the axioms as premisses and the theorem as conclusion. Unless the theorem is a trivial consequence of the axioms, this is not what we mean by a proof.

The concept of proof is epistemic and can obviously not be reduced to a non-epistemic concept of valid inference. The question I am raising in this lecture is if we can account for this epistemic nature of the concept in a less metaphoric way than saying “is seen to” and if proofs can then come out as something objective. The distinction between an inference being evidently valid and being merely valid was made already at the birth of logic when Aristotle distinguished between what he called perfect and imperfect syllogisms, but he did not explain the distinction any further. Nor does our modern idea of formal proof contribute to such an explanation.

The more general concept of ground as used in epistemology and philosophy of language is closely connected with the concept of proof. A speaker is expected to have some grounds for what she asserts, and an assertion is evaluated as justified or warranted when the speaker has a sufficiently strong ground for the assertion. Here I restrict myself to the strongest possible grounds, what we call conclusive grounds, which should not be confused with infallible procedures for arriving at knowledge. We conceive of deductive proofs as one way in which we can obtain conclusive grounds; in the sequel I drop the attributes “deductive” and “conclusive”, but always mean deductive proof and conclusive ground when I say just proof and ground.
What constitutes a ground for an assertion obviously depends on the meaning of the assertion. Here I shall take the view that the meaning of a proposition is given in terms of what is considered to be a ground for the assertion that the proposition is true.

In this context one should consider the notion of proof that occurs in the intuitionistic interpretation of logically compound propositions, often referred to as the BHK-interpretation. The question what a proof of an implication \( p \rightarrow q \) consists in has however been controversial in this tradition. According to Heyting (1934) it consists of a construction \( c \) that transforms any proof of \( p \) into a proof of \( q \). It may be objected that one may have a construction \( c \) that in fact transforms any proof of \( p \) into a proof of \( q \) without being able to recognize that \( c \) has this property, and that in such a case it would be strange to call the construction a proof of \( p \rightarrow q \). For this reason Kreisel (1960) suggests that a proof of \( p \rightarrow q \) is a pair \((c,d)\) such that \( c \) transforms any proof of \( p \) into a proof of \( q \), and \( d \) proves that \( c \) has this property. But this is to bring in what a proof is in the explanation of the concept so as to make the explanation loose its recursive character.

In his type-theory, Martin-Löf avoids this controversy by taking the proofs spoken of here to be mathematical objects without epistemic significance; such significance arises in his theory first in connection with a *demonstration* of a fact that a “proof object” is of a specific type.

I shall here reinterpret the proofs of the BHK-interpretation as epistemic objects, not as proofs but as grounds for assertions. This makes it possible to regard an inference as essentially an operation on given grounds for the premisses; the inference is valid, if the operation yields a ground for the conclusion when applied to the grounds for the premisses. We may then consider the proposal to take proofs to be chains of such valid inferences. They are objective but have nevertheless epistemic significance, since making a chain of inferences, in other words carrying out the operations in question, one comes into possession of a ground for the conclusion of the last inference of the chain. A proof in the proposed sense is thus describing a construction process which results in a construction of a ground for the final conclusion of the proof. By going through this process, one gets aware of a construction of a ground for this conclusion.

One may object to the idea of calling such a chain of valid inferences a proof, pointing out that a subject who makes these inferences certainly gets aware of a construction which in fact is a ground for the conclusion, but may nevertheless lack awareness of this fact. To support this objection one can argue that a chain of valid inferences, also in the sense now proposed, is not a proof unless the fact that it delivers a ground for the conclusion is recognized or can be recognized; the question may then again arise whether to be a proof is something subjective. To evaluate this objection, one has to ask whether it amounts to saying that in order for an assertion to be justified it is not sufficient that the speaker is in possession
of a ground for the assertion: she must also establish that the ground that she is in possession of is a ground for the assertion in question. Such a demand would obviously lead to a regress and should be rejected, but I leave open whether that demand really forms part of the objection.

Mme Renate Schlesier
Professeure, Freie Universität, Berlin (Allemagne)

Dionysos et la 36e question grecque de Plutarque

Laissant de côté une autre occurrence du « pied » de Dionysos (chez Sophocle, Antigone, v. 1143), la conférence se concentrait sur la 36e Question grecque de Plutarque dont le début est le suivant : « Quand les femmes des Éléens implorent Dionysos par un hymne, pourquoi l’engagent-elles d’apparaître parmi elles avec un pied de bœuf ? » Cette question, l’hymne lui-même (dont Plutarque est le seul témoin) cité à sa suite et le commentaire de Plutarque à son sujet, sont mis en rapport avec les livres 5 et 6 de la Description de la Grèce que Pausanias consacre à la région d’Élide. Or, par opposition à d’autres lectures, les résultats de l’analyse présentée au cours de la conférence sont les suivants : 1) La déclaration de Plutarque que « les femmes éléennes » chantent cet hymne à Dionysos, est à prendre à la lettre ; il ne faut pas identifier les femmes éléennes au corps des seize femmes de l’Élide (qui est distingué d’elles chez Pausanias comme dans l’œuvre de Plutarque). 2) Les obligations culturelles des seize femmes et celles des femmes éléennes en général sont différentes ; selon Pausanias, les différences concernent notamment des actes divers dans le contexte du culte des héros. 3) La référence topographique exprimée dans l’hymne (« temple auprès de la mer ») est à prendre au sérieux ; la conjecture « temple des Éléens » n’est pas justifiée. 4) Pausanias et Plutarque témoignent d’activités cultuelles destinées à amener l’épiphanie de Dionysos et exécutées, d’un côté, par un petit groupe de spécialistes religieux masculins pendant la fête des Thya (Pausanias), de l’autre, par une multitude de femmes au statut non différencié (Plutarque) à une occasion non spécifiée ; ces deux catégories d’actes cultuels sont localisés en des endroits différents de la région d’Élide, fortement éloignés l’un de l’autre (le premier aux alentours de la ville d’Élis, le deuxième au bord de la mer, à une distance considérable de ce centre urbain). 5) Cela n’exclut pas la simultanéité cultuelle du rite exécuté par des hommes et de la convocation de Dionysos par des femmes. 6) Dans leur hymne, les femmes éléennes qui appellent Dionysos à apparaître parmi elles demandent son épiphanie « avec