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Introduction

1 Until a few decades, studies of early maps were not always interested in metric accuracy, with the larger share of attention being devoted to the accuracy of the content. Although the planimetric accuracy of local aberrations was often determined, systematic quantitative assessments of the overall cartographic image were not seldom ignored. The reason for this should probably be sought not only in the fact that systematic measurements on dimensionally unstable documents – which early maps certainly are – usually take too much time and processing such data is a complex matter, but also the persons studying these maps are often historians – or persons with historic interests – who more often than not are sooner interested in content accuracy. Progress is made, especially since the publication of Kishimoto (1968).

2 Thanks to the computer facilities and the evolution of the statistical methods, a more efficient accuracy approach is now within the bounds of possibility, even for maps without the expression of geographic coordinates. A pure mathematical similarity transformation on a significant selection of unambiguous points in the historic map gives us a good insight in the metric accuracy of the map. This is necessary to know the quantitative impact of other items in the map. The Procrustes method (see Mekenkamp, 1986) is for this purpose a ideal instrument. Several other methods were used in the past; a short overview will give us a good synthesis.
Materials and methods

Early maps with known grid system

The method used to evaluate metric accuracy in early maps depends on the specific characteristics of the map in question. The size of the charted area and the scale, of course, not only affect the absolute accuracy but also play a role in the decision whether to take into account the curvature of the earth. The last decision becomes unnecessary if the map projection system is (explicitly) mentioned. In this case, it is possible to fix the position directly, which greatly simplifies the required mathematical processing (A. Strang, 1998). The geographical co-ordinates can be derived from the map edge data for longitude and latitude, or based on the pattern of meridians and parallels, which occasionally run through the entire map area. The noted values can then be compared directly with the actual geographic co-ordinates, which in turn can be derived from the current topographic map.

This double set of co-ordinates (obtained from the “early” and from the “current” map) can be statistically processed in order to establish clearly the metric accuracy of the mapping (already in 1931 by Jacyk). For example, one can calculate correlation coefficients or construct frequency diagrams for different (categories of) deviations; one may then choose to determine the trend and/or draw up cumulative curves of the errors, etc. Another possibility is to calculate the average, the largest and smallest deviation. A further option would be to check whether these deviations are distributed uniformly over the entire map area (S. Pietkiewicz, 1960). Theoretically, the deviation can be determined for every point on the map, provided that this point can be related to a currently existing homologous point.

In the event that the map lacks a reference frame (metric edge data on the map), one can use known geodetic points, if available, or “probable” triangulation points. With this we mean: clearly visible landmarks that correspond to current higher-order triangulation points, on which the reference frame or grid can be constructed, the same method that was presumably also employed in the past, although this is not explicitly recorded. Thus, it is possible to convert the map co-ordinates of all other points to this system (figure 1).

It is known that the topography of many early maps (16th – 17th century) is restricted to a selection of planimetric features, such as village and town centres, occasionally supplemented by basic hydrography. Because historic river courses cannot always be compared to modern courses, and also because they were often depicted only schematically on the map, it is obvious that studies of the accuracy of such maps are restricted to evaluating the positioning of settlements. Often, they are represented on the map by a small symbol. Consequently, locating them is relatively simple and unambiguous.
Figure 1. An X-Y-grid can be constructed on the early map, that lacks a reference frame, starting from known or “probable” triangulation points. Then, a similarity transformation will be executed on these three points and the corresponding points on the actual topographic map. All other identified points on the historic document can now be linked to the actual ones in the existing grid.

Early modern maps: To what extent are they metrically accurate?

The situation is very different in early maps that lack a planimetric reference frame and even geodetic points. In such cases, one has to refrain from absolute quantitative comparisons. If, in addition, the substrate – often paper or parchment – is all but dimensionally stable and has been subjected to irregular deformations over the centuries, analyses become much more complex. In such situations one has no choice but to restrict oneself to relative measurements. In most cases this will suffice for satisfactory answers.

One of the historic methods that is employed to determine the relative accuracy of the positioning of points on the map derives from the ratio of distances between each mapped point to all other mapped points (figure 2) which is compared, in turn, to the corresponding distances and actual distance ratios (R. Kirmse, 1957; J. Szeliga, 1967; J. Stone & A. Gemell, 1977). Although these measurements and calculations are completely scale- and orientation-independent; they are very extensive, when doing these by means of the analogue procedure. An example makes this clear: for a map depicting only 100 points, one has to measure and possibly compare nearly 5,000 distances. After this, the same number of measurements must be carried out on the corresponding map and subsequently compared to those made on the early map. A wide range of statistical operations can be performed on this data, providing a quantitative impression of the extent of the deviations. The same method can be applied to any other map, thus making it possible to compare different maps as regards their deviation parameters (J. Hooke &
R.A. Perry, 1976). Computerised processing of this data is obviously much more rapid and will be discussed below.

Figure 2. To compare historic map points to the corresponding actual items, some authors started from the equation of the mutual distances.

A different method (construction of a distortion grid) that also has been mentioned in the literature for a relatively long time, not only provides numerical results (quantitative method). It can also give us a visual impression of the size of the deviations and a global picture of the deformations (qualitative method). A grid or a geographic graticule is placed over the current map and transferred to the early map, or vice versa. To this end, one establishes the position of every intersection on the deformed map, based on at least three nearby points that are also found on the corresponding map (figure 3). Now, as in the previous cases, one can compare the locations of the homologous points and process the results statistically. Again, one can then make statements about accuracy, expressed as correlation coefficients or as average deviations per km, either for the entire map or for homogeneous partial areas (R. Schmidt, 1976; F. Depuydt, 1990 and G. Forstner, 1998).

For instance, with the help of this method, we have calculated relative metric accuracies in percentages for the national historic mappings by Ferraris (1770-1777), the “Militaire Verkenningen” (1815-1830), and Vandermaelen (1846-1854). For informational purposes we again list the difference or error percentages for these three periods: 10%, 7.5% and 5%. In practice, this means that the sides of a square grid that originally measured 1 km by 1 km, can vary between 900 and 1,100 m, between 925 and 1,075 m, or for Vandermaelen between 950 and 1,050 m, respectively. This, naturally, was based on the assumption that the present topographic map is accurate, or in other words that the error falls within the cartographic mapping accuracy, i.e. in this example ± 5 metres.
Figure 3. One can reconstruct a grid or graticule from the topographic map on the corresponding early map, starting from reference points in the immediate vicinity of the intersection point of the grid. The mutual differences in position of the homologous point can now be processed statistically.

The listed error percentages may appear rather large compared to the 3.4% relative accuracy of Mercator’s map of Flanders, as calculated earlier by R. Kirmse (1957). One should realise, however, that all the measured points on the Mercator map are triangulated points that – due to the nature of the mapping – have been measured much more precisely than any other random points, on which we based ourselves to construct the transformation grid on the maps by Ferraris, “Militaire Verkenningen”, and Vandermaelen.

Improved analysis thanks to computers

When digitising corresponding sets of points on the early and current map, it is possible to perform a similarity transformation and process the results statistically. This transformation, in fact, consists of attempting to match the set of historic points as closely as possibly to the “current” cluster (K. Brunner, 1995). This can be accomplished in different ways (H. Kishimoto, 1968 and W. Ravenhill & A. Gilg, 1974). We reduce the following review to a minimum.

A first approach is partly based on one of the previous methods (J. Szeliga, 1967; J. Stone, 1993 among other), in which one compares the mutual distances and then stretches one of the clusters so that the sum of squared differences is minimal. In doing so, one can choose to consider all the selected points (J. Stone & A. Gemmel, 1977) or to make a limited selection by first eliminating the greatest errors (P. Mekenkamp & O. Koop, 1986 a.o.). The advantage of the latter method is – as mentioned previously – that accidental “major errors” do not affect the rest of the cluster. Mekenkamp took the process a step further with his “Standard inaccuracy circle method”. In each point a circle is drawn whose diameter is proportionate to the deviation (figure 4), although this value does not correspond directly to the absolute displacement of the point on the early map (see also K. Brunner, 1995). After performing an initial calculation, Mekenkamp draws a standard inaccuracy circle in function of what he terms the point deviation value or “d-value”. Then, he again calculates the least squared average deviation for the remaining points and again eliminates the point with the greatest deviation, after first having drawn the corresponding deviation circle or “inaccuracy circle”. He continues these iterations until
Building on the mentioned iterative method, we have applied a different method of transforming co-ordinates, which also made it possible to represent the absolute deviation per point and simultaneously indicate the direction of the displacement. We performed a “Procrustes analysis” (T. Cox & M. Cox, 1995). It is a simply similarity transformation and here we added the iterative steps. So we did the transformation first on all points and then again after each iterative step. In other words, the historically mapped cluster is translated, rotated and the scale is adjusted (dilated) at each iteration until all “major and random” errors are eliminated from the system. As the last step of this iterative Procrustes analysis, in which the relative map co-ordinates of the remaining points were calculated also, the transformed co-ordinates of the earlier eliminated points also will be calculated. The planimetric differences between corresponding points of the historic map and the present-day location are then drawn in true size (albeit adjusted to the appropriate scale) as the radius of -what we mentioned- a “deviation circle”. This results, as with Mekenkamp’s method, in a clear picture of the local and global deformations of the map, or rather: a picture of local inaccuracies, from which an average value can be extracted for use in further statistical processing.

The accompanying figure 5 shows the different steps of this process:

1. The centre of gravity of both clusters is calculated (one for the early map and one for the corresponding points on the current topographic map), whereby X (and Y) is the average of all X (and Y) values in all points.
2. From these two central points, all compass directions to the remaining points are summed up for each of the two maps. Finally, the historic mapping is rotated so that it is oriented in the same average direction.

3. Both clusters can then be superimposed according to their centre of gravity and average direction.

4. The scale of historic points is adjusted or stretched by equalling the sum of the distances from each point to the centre of gravity with that for the corresponding modern topographic map. Note that the steps 2, 3 and 4 are implemented in a integrated way and not sequentially.

5. We then determine the deviation of each (historic) point from its current location. In fact, the Procrustes analysis ends at this stage. From here, our new approach will be explained. The point with the greatest deviation is no longer considered (eliminated from the series) when repeating all previous steps of the Procrustes analysis. Thus, the same process is repeated for the remaining points (i.e. all points minus one). Again, the point with the greatest deviation is left out of the series and the process is repeated. This could be continued until only one point pair remains. Once the last calculation is completed, the new situation of each historic point is transformed and determined. For each point, the deviation from the current situation and its direction are represented by means of respectively a circle and its corresponding radius. More extensive explanation can be found in F. Depuydt a/o, 2000.

15 All these calculations obviously consume relatively large quantities of computer time. This is partly why, in practice, the iterations are only carried out there where the deviation curve or error-distribution curve displays a dip between the first "rough and excessive" deviations and other erroneous displacements (cf. F. Depuydt, 1998).

16 As before, further correlations and/or statistical operations can be performed on these results, making it possible to compare historic mappings in their entirety.

**Some applications**

17 Using this iterative Procrustes analysis method, we can now check the accuracy of the original surveying. One can even establish the precision with which copyists of historic maps performed their work. Naturally, it is also interesting to learn, for example, the reasons for certain large local deviations or for certain trends in the measurement errors, or for particular constant directions or sizes of planimetric displacements... It is even possible, for instance, to determine the accuracy in locating historic items that have disappeared, such as old sites.

18 Few cartographic documents from the 16th or 17th centuries mention the origin of the survey. Copies from this period, too, often lack any references to their origins. The examples analysed below date from the earliest days of triangulation usage in land surveying. These maps are not only an original and a copy whose origin is known, but also a map from the same period whose original is not conclusively established.

19 A study of the metric accuracy of the *Map of Flanders by Gerardus Mercator* (1540), should give us an idea of mapping accuracy in the early phases of triangulation use on land. On the basis of the copy made of it by *Abraham Ortelius* (and who mentioned it explicitly) for his *Theatrum Orbis Terrarum* (1570), it is possible – thanks to the Procrustes analysis method – to determine the accuracy of the copy, to establish the links between both documents and to reveal the differences between both maps. Also created around the
same period was the map of the *Brugse Vrije* (merely a part of the above-mentioned maps, i.e. ~16%) It was painted by *Pieter Pourbus* (1571) and it is not known to which extent he relied on the previously mentioned documents. Although the three maps are executed on a different scale, the accuracy results are reliable because the graphic deviations in the studied maps exceed to a great extent the graphic accuracy (an average of about 10 times).

Figure 5. The Procrustes analysis is a mathematical similarity transformation. The iterative phases are recalculations of the similarity transformation after omission, at every turn, of the point with the largest erroneous aberration from the correct topographic position. The consecutive steps in the iterative Procrustes analysis are clearly reflected by this figure: first of all a translation of the sets; then an integrated rotation and dilatation of the pattern of point locations on the historic document in proportion to the corresponding points of the topographic map. This analysis will be redone each time for the same cluster, minus the most aberrant point. So the largest accidental errors will not influence the final similarity transformation and thus the most optimal fitting of the two point clusters will be realised.
Figure 6. Illustration of the inaccuracy of the Flanders map of G. Mercator. The length of the radius of the inaccuracy circles corresponds to the absolute mapping deviations of the more than 1000 settlement symbols on the map; the sporadic drawn radius direction on the other hand to direction of the displacements.

Exactissima Flandriae Descriptio by G. Mercator: a satisfactory triangulation?

The circles in figure 6 represent the proportional mapping deviations of all church spires of the towns and villages listed on Mercator’s map of Flanders from 1540. The absolute size of the local errors and the direction of the displacements correspond with the size and direction of the radius, which is drawn to scale. The accompanying table (figure 7) gives the average values of the deviations (both on the map and in reality) for 913 of the 1017 points that were studied (i.e. about one hundred points cannot be located on current maps!). More detailed figures and a list of historic and corresponding modern toponyms can be found in an earlier publication (F. Depuydt, 1998).
A first noteworthy fact is that the central area of the County of Flanders and particularly the Waas region ("Land van Waas") clearly are mapped more accurately than the remaining areas. Deviations in the Waas region, for example, never exceed 150 metres, that's nothing compared to the overall average deviation of 815 metres for the entire map (and smaller than the graphic symbol!). The edges of the area and particularly the border areas with France have a notably greater deviation; so they are clearly less accurate.

A second intriguing fact is the preference in the direction of deviations in certain regions, such as the northern coastal area between Nieuwpoort and the Scheldt, and both south-easterly lobes in the direction of Beveren and Valenciennes. This directional preference is expressed in the direction of the depicted circle radius. A probable source is an accumulation and propagation of measurement errors made in the field (or while engraving??). Explanations and hypotheses for this and other examples can be found in the above-mentioned article (F. Depuydt, 1998).

**Flandria by A. Ortelius: a copy?**

The Flandria atlas map by Abraham Ortelius, which he made at slightly less than half the scale used by Mercator for his map, is not a perfect copy, as he left out or added several dozen toponyms (cf. table). Even at first glance, one clearly notes the resemblance of the deviation circles (figure 8) to those of the Mercator map. The increased deviation from the current topographic map (average deviation of 1,020 m) would then be a logical consequence of the copying. From the table (figure 7) we learn that town and village symbols were copied at an accuracy of 1.7 mm. Thus the accuracy (cf. table) with respect to Mercator's original is twice as good (650 m) as with respect to the current topography (1,020 m). The average copying accuracy, understandably, is most pronounced in the Waas region, where Mercator used very precise results, i.e. the circle diameter for Mercator was virtually zero, while that for Ortelius approximately equals his average copying precision, i.e. about 650 m.
24 In the Procrustes analysis, the degree of correspondence or correlation between the respective maps is expressed by what Cox and Cox termed the Procrustes statistic ($R^2$). $R^2$ expresses the level of inaccuracy as a figure varying from 0 to 1. At a value of 0 both clusters fit perfectly, while at a value of 1 there is no logical connection whatsoever. The relatively low values in the table indicate a large measure of correspondence, whereby it is strongest for Ortelius in relation to Mercator; it is twice as good than Mercator in comparison to the contemporary topographic map and nearly three times better than Ortelius compared to modern topography.

25 The comparison of the metric accuracy of both historic mappings could be shown in a figure that superposes the two results (respective accuracy against current topography) but is shown here in an enclosed figure (figure 9) comparing the planimetric displacement of villages between the maps of Ortelius and Mercator. As expected, the circles of relative deviation have shrunken considerably. This, then, is a graphically and spatially distributed illustration of the figures from the table.

26 Ortelius added about 40 new points to those given by Mercator. The average error for these is nearly twice as large as for other locations, and nearly three times the results for Mercator. The dozen points that were left out are coincidental. This is because these are not Mercator’s less-accurately recorded points that were eliminated, nor points in areas with strong concentrations. No more than a few are located in the area of the Scheldt polders, which were often inundated, so it is possible that these settlements could have disappeared.
The “Brugse Vrije” by P. Pourbus: another copy?

27 This large-scale map (1:12,000) painted in 1571 of the “Brugse Vrije”, copied by P. Claessens in 1601, contains about a 105 toponyms that were also included by Mercator. The question arose to which extent Pourbus was familiar with Mercator’s work and whether he copied him in 1560, when he first started his work, and – if this were not the case – whether its metricity approaches or exceeds that of Mercator’s map.

28 To confirm or reject the hypothesis, the results of this study – which were previously published (F. Depuydt & J. Theelen, 1998) – are best compared with those of the just mentioned study results.

29 The Procrustes analysis was applied in a similar way, namely by studying the accuracy of the Pourbus map compared to the current topographic map, and by correlating between the maps of Pourbus and Mercator. The results can be followed either on the maps (figures 10, 11, and 12) or – for a more global overview – in the table (figure 7).
Figure 10. Local deviations of the settlement symbols on Mercator's map for the region of the Brugse Vrije. The deformation radius is only expressed for the locations mentioned in the article.

Figure 11. Accuracy circles for the mapped settlements of the Brugse Vrije on the painting of Pieter Pourbus and his perfect copy by Pieter Claeissens.
Results and discussion

Contrary to the preceding conclusions, we note that Pourbus achieved a considerably higher accuracy than Ortelius or even Mercator, namely an average absolute accuracy of 340 m. This is 25% more accurate than the results of Mercator for this smaller area (480 m)! The greatest deviations, although these are located at the borders of the County, are not found in the same locations as for Mercator. Some examples: the eastern zone of the correlation map (Pourbus vis-à-vis Mercator, figure 12) has a uni-oriented deviation, mostly to the NE; the settlements of the SW-region have a difference in situation of about 1.5 to 2 km mainly! The numerical and visual comparisons between Pourbus and Mercator, too, show lower correlations than between each of them and the current topographic map, i.e. a mean deviation 550 m against 340 m and 480 m, respectively.

The contrast with our observations for the above comparison between Ortelius and Mercator is striking (650 m against 1,020 m and 815 m !!). Consequently, we can claim with relatively great certainty that Pourbus had his own sources, which very probably were based on comparatively correct triangulations.

More detailed analysis and explanations can be found in the above-mentioned study (F. Depuydt & J. Theelen, 1998).
Conclusion

We exposed here an iterative application of the Procrustes-similarity transformation to early maps. This method not only teaches us much about their relative and absolute mapping accuracy, but also about relationships between different mappings of the same area. It has also objectively shown that it makes it possible to obtain an impression of the accuracy with which past cartographers copied and published each other’s maps.

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**ABSTRACTS**

A mathematical similarity transformation is often applied to the study of metrical accuracy of early maps. Accidental major errors of some points in the early map affect the mean accuracy index of the map to a large extent. Therefore, here we propose to exclude these major errors until we made the similarity transformation. Finally we calculated the local inaccuracies of the cluster points. At this rate we obtain a more realistic and optimal expression of the map deformation. The latter is illustrated by some early maps of the County of Flanders. The Pourbus map (1571) becomes a much higher accuracy result (340 m) in comparison with the Flanders map of Mercator (1540) and Ortelius (1570) having an accuracy of respectively 815 m and 1,020 m.

On utilise souvent la transformation de similarité pour étudier la précision métrique des premières cartes. Il arrive que des erreurs accidentelles majeures se produisent sur certains points, entraînant des conséquences déterminantes pour l’indice de précision moyen de la carte. C’est pourquoi nous proposons d’éliminer ici ces erreurs majeures jusqu’à l’accomplissement de la transformation de similarité. Nous avons finalement calculé les inexactitudes locales des points clusters. Ainsi nous obtenons une expression plus réaliste et optimale de la déformation de la carte, comme illustré dans plusieurs cartes anciennes du Comté des Flandres. Ainsi, pour la carte de Pourbus (1571), nous arrivons à un résultat bien plus précis (340 m) en comparaison de la carte de Flandre de Mercator (1540) et de celle d’Ortelius (1570), pour lesquelles la précision est respectivement de 815 m et 1020 m.

De metrische nauwkeurigheid van oude kaarten wordt dikwijls bepaald door toepassing van een wiskundige gelijkvormigheidstransformatie. Jammer genoeg is hierbij de invloed van ‘toevallige grove fouten’ soms dominerend, waardoor het totaalbeeld van de historische precisiemeting merkelijk wordt scheefgetrokken. Aan de hand van de statistische Procrustes-methode is het makkelijk de uitzonderlijk grote toevallige fouten uit te schakelen en een realistischer beeld te geven van de karteernauwkeurigheid. Dit wordt geïllustreerd aan de hand van enkele bekende oude Zuidnederlandse relatief grootschalige karteringen van het Graafschap Vlaanderen uit de 16de eeuw zoals de kaarten van: Mercator (1540), van Ortelius (1570) en die van Pourbus-Claei sens (1571-1601), die een karteernauwkeurigheid etaleren van respectievelijk 815 m, 1,020 m en 340 m.
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Mots-clés: précision des cartes, précision métrique des cartes anciennes, carte de Flandre de Mercator, carte des Flandres d’Ortelius et Claeissens (“Brugse Vrije”)
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