Reasoning from or reasoning about beliefs: Truth-based or possibility-based compatibility judgments and Handley et al.’s (2006) litmus test of the suppositional conditional

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Introduction

Much of our knowledge is hypothetical in nature and can be expressed in the form of conditionals (e.g., If you study hard, you will succeed in the exams) expressing the contingency between an antecedent (A) and consequent (C) event. We can use such knowledge or beliefs to guide our actions (e.g., given a desired outcome like succeeding in one's exams, one knows what to do). In such cases one is reasoning from a particular belief: one takes one's beliefs for granted and does not put them into question. In other cases, the beliefs itself are put into question: one is reasoning about a particular belief. Is it really the case that studying hard will ensure succeeding? Is it possible that one can succeed without studying hard? Reasoning about our beliefs seems particularly prevalent when expectations do not pan out or when integrating new observations and experiences within our belief system. A desirable property of such a belief system seems to be its internal consistency, even though many people will often not be concerned with the coherence of their beliefs and will hold mutually incompatible beliefs. How do people go about in evaluating the consistency of their beliefs? Do people reason from or do they
reason about their beliefs in judging whether they are compatible, or both? The present paper addresses this question via the theoretically loaded discussion regarding the compatibility of two conditional beliefs of the form <if A then C> and <if A then not-C>. We refer to such conditionals, which most people consider contradictory, as “contrary conditionals”.

2 Handley, Evans and Thompson (2006) presents a thought provoking analysis of <if A then C> conditionals, whose believability/acceptability in suppositional-conditional theory is a function of the conditional probability of the consequent <C>, given the antecedent <A>. When people reason about rather than reason from conditionals (i.e., when they do not accept the conditional as true and question its plausibility/acceptability), having a conditional-probability interpretation of "if it rains, then the match will be cancelled" means its plausibility is measured as the conditional probability of the match being cancelled, given that it rains. This conditional probability is a function of the relative likelihood of <A_C> and <A_not-C> cases: e.g., situations in which a match is cancelled when it rains and situations in which a match is not cancelled when it rains (see, e.g., Evans, Over, & Handley, 2003; Schroyens, Schaeken, & Dieussaert, 2008). Classically these two so-called truth-table cases are referred to as true-antecedent cases; respectively:

- True-antecedent-True-consequent: TT<A_C>
- True-antecedent-False-consequent: TF<A_Not-C>
- False-antecedent-True-consequent: FT<not-A_C>
- False-antecedent-False-consequent: TF<not-A_Not-C>

These false-antecedent cases are considered irrelevant according to the suppositional-conditional theory and its conditional-probability approach to reasoning about (vs. from) conditionals.

3 The presumed irrelevancy of false-antecedent cases in suppositional theory stands in contrast to mental-models theory. Like suppositional theory, mental-models theory supposes people first consider only true-antecedent cases. More specifically, it supposes initially only TT<A_C> cases are considered. This is the so-called implicit-model principle (a.k.a. the initial-model principle: Johnson-Laird & Byrne, 2002; Schroyens & Braem, in press). The true-antecedent case TF<A_Not-C> is not initially considered according to the so-called truth-principle (cf. Johnson-Laird & Byrne, 2002, p. 653; Schroyens, Schaeken, & d’Ydewalle, 1996, 1999). The truth-principle concurs with a Gricean maxim of conversation (Grice, 1975): we always begin by assuming the truth of given sentential information. Since falsity cannot exist prior to truth (just as negation cannot exist without something to be negated), it seems no theory can avoid making such an assumption. Due to the initial-model principle, and the false-antecedent cases not being represented initially, the false-antecedent cases will tend to be judged irrelevant. But, the irrelevancy is not fixed. It holds initially, i.e., it holds when making judgments on the basis of the initial-model representation. Alternative possibilities to the initial-model are considered and mental-models theory maintains that irrelevancy judgments of such alternative false-antecedent cases are dependent upon the specific content of, and the context of reasoning from or about, conditionals (cf. the principles of pragmatic and semantic modulation; Johnson-Laird & Byrne, 2002, p. 658-659).

4 As a consequence of the initial-model principle and the principle that at least some people at least sometimes consider alternatives, it is only at first sight that false-antecedent cases are judged irrelevant. When people do not satisfice, pragmatics and semantics aid in constructing alternatives to this initial model of TT<A_C>. For instance,
when asked to evaluate false-antecedent cases like FF<Not-A_Not-C> (e.g., it does not rain and the match is not cancelled), the pragmatic context gives a clear reason to consider FF<Not-A_Not-C> cases: one is asked to do so, and even if one were unwilling to participate in the task, the mere fact of having read the question almost forces consideration of FF<Not-A_Not-C>. That is, when there is no need to generate FF<Not-A_Not-C> (since provided by context), people have only to judge whether it is consistent or inconsistent with <if A then C>. Mental-models theory proffers only TF<A_not-C> falsifies <if A then C> (cf. the core-meaning principle: Johnson-Laird & Byrne, 2002). It follows that both FF<not-A_not-C> and FT<not-A-C> (e.g. it does not rain, but the match is nonetheless cancelled, e.g., because of an extreme heat wave) are possible, given the truth of the conditional.

5  FF<Not-A_Not-C> being a true possibility is central to the Handley et al (2006) critique of mental-models theory. (True possibility is short hand for a contingency that is possible, given the truth of the utterance and does not mean such a case makes the conditional true: cf. infra, also see Schroyens, 2009). The paper argues that <if A then C> and <if A then not-C> are incompatible according to suppositional-conditional theory and compatible according to mental-models theory and presents compelling evidence that in fact, most people do not consider these contrary claims compatible (also Pollard & Evans, 1980). If and only if their argument and treatment of mental-models theory is right, they provide a strong case against mental-models theory and establish an increase in suppositional theory's degree of corroboration. However, a close reading of Handley et al. (2006) reveals there might be a problem with the argumentation and treatment of mental-models theory presented therein.

6  Handley et al. (2006) supposes that a task requires reasoning from conditionals when they are evaluating their own theory while they suppose a task requires reasoning about conditionals when they are evaluating an opposing theory. Contrary conditionals are incompatible when one assumes – as Handley et al. (2006) claims mental-models theory does – people engage in the following type of compatibility-judgment processes:

7  Two conditionals are compatible in the model theory because they share two common possibilities: ~pq and ~p~q" (Handley et al., 2006, p. 560). The use of possibility-based compatibility judgments presumes people are reasoning about the truth or truthfulness of conditionals on the basis of possibilities, whereas suppositional-conditional theory only predicts the incompatibility of contrary conditionals when it is assumed people are reasoning from the presumed truth of conditionals. "In the extreme case where one of these is assumed certain (P (q/p) = 1), then the other must be false (P (¬q|p) = 0)" (Handley et al., 2006, p. 559). A task obviously does not suddenly change in the eyes of the participants depending upon the theory an experimenter applies to explain their performance.

8  Handley et al. (2006) seems to adopt a different task construal and different notion of compatibility to judge their own theory as compared to what they present as a rival theory -- which for one or the other reason they presume needs to make opposing predictions (see, p. 560: “Clearly, the account cannot agree”), even though it is clear that such is not necessary: most theories that have some plausibility will show considerable overlap in the phenomena and established facts that they can and need to explain. We can altogether set aside or postpone a further theoretical discussion as regards the at least apparent case of special pleading in Handley et al. (2006), which is likely to be of more interest to experts and/or more philosophically interested readers who are
concerned with the fairness, rigor and correctness of scientific argumentation. Possibility-based compatibility is ultimately a hypothesis about how people go about in reasoning about two conditionals in order to decide whether they are compatible or not. It might ultimately be a mistake to assume that people are reasoning about the two conditionals to ascertain whether they are compatible. Maybe they engage at least in part in reasoning from their beliefs in order to judge whether they are compatible. Let us start our investigation by considering the former possibility both within mental-models theory and suppositional-conditional theory.

**Leveling the bar: possibility-based compatibility judgments**

In Handley et al. (2006) we see the theoretical statement that – according to Handley et al.’s interpretation -- mental-models theory predicts that contrary conditionals are compatible.

“According to the extensional semantics of Johnson-Laird and Byrne (2002), the conditionals [if A then C] and [if A then not C] are not incompatible” (Evans et al., 2005, p. 1049). Given the presumed possibility-based compatibility-judgment process, it is indeed easy to see that in mental-models theory <if A and C> and <if A and not-C> should sometimes be judged compatible. At best one can make the prediction that only sometimes the contrary conditionals would be compatible. Mental-models theory makes several processing assumptions that prevent making the prediction that all people judge contrary conditionals to be incompatible, even when we accept the assumption that people engage in possibility-based compatibility judgments. The assumption that people engage in making possibility-based judgments is a real assumption. This assumption is not a given, even though Handley et al. (2006) suggests it is a given about reasoning within mental-models theory.

**Table 1 Truth contingencies that are possible for two contrary conditionals, when reasoning from the assumption the conditionals are true.**

<table>
<thead>
<tr>
<th>Conditional: if p then q</th>
<th>Contingency if A then C</th>
<th>if A then Not-C</th>
</tr>
</thead>
<tbody>
<tr>
<td>TT(p.q): A C</td>
<td>A C</td>
<td>A Not-C</td>
</tr>
<tr>
<td>FT(¬p.q): Not-A C</td>
<td>Not-A C</td>
<td>Not-A Not-C</td>
</tr>
<tr>
<td>FF(¬p.¬q): Not-A Not-C</td>
<td></td>
<td>Not-A Not(not-C)</td>
</tr>
</tbody>
</table>

Mental-models are in essence representations of possibilities and mental-models theory proposes that one’s interpretation of a natural language sentence is determined by the possibilities one considers consistent with the sentence. If people go beyond the initial-model representation (i.e., TT<A_C>) and if people reason towards the so-called conditional interpretation of the contrary conditionals <if A then C> and <if A then not-C>, then people will consider three possibilities that are consistent with the assumption that these conditional are true. Table 1 presents the three true possibilities, that is, the contingencies that are possible when reasoning from the conditionals by assuming these
beliefs are true. Considering these two model sets, it is clear people would consider <not-A_not-C> cases true possibilities for both <if A then C> and <if A then not-C>. The <not-A_not-C> cases are instances of FF vis-à-vis <if A then C> and they are instances of FT vis-à-vis <if A than not-C>. Both FF and FT cases are true possibilities when people reason towards a conditional interpretation. Hence, the conditionals are* not possibility-based incompatible -- at least for those people who do not satisifice with their initial representation of such conditionals and do not reason towards an alternative interpretation such as the biconditional interpretation. We use * here as a reminder of the slippery slope between the prediction that something 'has to be the case", i.e., is a factual necessity as opposed to 'can be the case", i.e., is a contingent possibility depending on other factors like to two we just detailed.

To follow up the consequence of a possibility-based compatibility-judgment process in mental-models theory, it is only required that the theory allows for shared possibilities in the representation of the two contrary conditionals. Stripping off much of the complicated and complicating theoretical baggage, it was easy to show that contrary conditionals are* not incompatible according to mental-models theory. It is just as easy to see that contrary conditionals are* possibility-based compatible in suppositional-conditional theory. This follows from the fact that suppositionalconditional theory claims, like mental-models theory does, that both FF and FT cases are possible when judging these cases as possible/impossible given the conditional it true (i.e., when reasoning from the conditional). If the possible nature of false-antecedent cases is sufficient to judge the conditionals compatible (as assumed by setting aside the additional requirements in mental-models theory that people do not satisifice and do not reason towards a biconditional interpretation: hence the “*”), then it follows that contrary conditionals are* also possibility-based compatible in suppositionalconditional theory. The theory supposes that FF and FT are possible cases. Hence, the contrary conditionals are* possibility-based compatible.

Suppositional-conditional theorists might rebut that the possibility-based notion is incompatible with the theory since it requires a truth-functional treatment of possibilities, which the theory does not support. That is, it would require that the FF<not-A.not-C> case makes the conditional true and that the same <not-A.not-C> case for if A then not-C (which now has the status of an FT case) similarly makes the contrary conditional true. Suppositional-conditional theory indeed holds that false-antecedent cases are irrelevant. However, this does not mean false-antecedent cases are never taken to make a conditional true. Indeed, meta-analyses on truth-table tasks – tasks asking people to judge whether the TT, TF, FT, FF case make the conditional true, false, or whether they are irrelevant (Schroyens, in press a) – have firmly established that people in fact frequently evaluate the false-antecedent cases as making a conditional true. This is a phenomenon that suppositionalconditional theory also proclaims to explain. It follows that nothing in the theory prevents it from considering a possibility-based compatibility judgments. If not, it would not be able to account for actual truth-table task performance. In Experiment 1 we consider the appropriateness of using the possibility-based notion of compatibility.
**Experiment 1: The inappropriateness of possibility-based compatibility judgments**

Let us assume that the Handley et al. (2006) definition of possibility-based compatibility is normatively justified and that it is correct to assume that people are reasoning about their conditional beliefs when evaluating whether such beliefs are compatible. Both theories would thus agree people should judge contrary conditionals compatible, given the established fact people judge false antecedent cases as being possible. But, should is not would (Mandel, 2000). Nobody expects modal reasoners to live up to idealized standards of normative theories (Stanovich & West, 2000; Stanovich, 2004; Schroyens et al., 2008, Schroyens, in press b). Handley et al. (2006) provided compelling evidence showing people generally do not live up to the hypothesized norm of possibility-based compatibility judgments.

The normative appropriateness of particular standards of judgments has been investigated on the basis of the so-called understanding-acceptance principle (e.g. Stanovich & West, 1999, 2000). The understanding-acceptance principle reflects the idea that if people understand the normative basis for a particular response they will accept that response and behave accordingly. When people understand the normative basis for a particular response (like a possibility-based compatibility judgments) and do not accept the response, this argues against the normative appropriateness of the response. Stanovich and West (1999) pointed out that there are two ways to implement the understanding-acceptance principle as a research methodology. First, the gap between the descriptive and the normative should be smaller for those individuals who are independently classified as more 'intelligent'. More 'intelligent' people would be more likely to understand and thus accept the norm. It is this individual-differences approach that has been applied most intensively, but which has also been criticised most intensively – probably in part because of the somewhat elitist idea that 'smarter' is 'better'.

The second method and application of the understanding-acceptance principle is less evaluative and involves setting up conditions in such a way that people are in the ideal situation to come to an understanding. Schroyens et al. (2008) accordingly labelled this the idealization hypothesis, i.e., the "general principle that when we approach the idealization, e.g., by imposing contextual constraints that bring us nearer to the idealized language game, we should observe behavior that more closely approaches the strictures and implications of that language game" (p. 180). Presuming for arguments sake that possibility-based compatibility is normatively appropriate, mental-models theory only allows for such compatible judgments when people do not satisfice with their initial representation and reason towards a conditional interpretation in which both the FF<not-A_not-C> and FT<not-A_C> are considered true possibilities. The present experiment thus created a context in which people will consider (though not necessarily accept) alternative possibilities to the presumed initial-model representation. Moreover, this contextual manipulation was embedded in a test-retest design, where during the retest of the compatibility question, participants were explicitly informed what it means for two utterances to be possibility-based compatible.

Method. After having obtained informed consent, we asked 141 final year (17-19 year old) Montreal high-school students to judge whether two contrary conditionals ("if the figure
is a circle, then it is red" and "if the figure is a circle, then it is not red", translated from French) are "compatible" or "incompatible". We subsequently asked them to judge (counterbalanced in order) whether a "non-circle that is not-red" (i.e. FF<not-A_not_C) is "possible" or "impossible" when assuming <if A then C> is true, and whether a "non-circle that is not-red" (i.e, FT<not-A_not-C>) is possible or impossible when assuming <if A then not-C> is true. After having answered these two truth-table task questions, they were again asked to evaluate whether the same two contrary conditionals are compatible. Moreover, and most importantly, in the re-test question participants were explicitly told that "two assertions are compatible when they have at least one possible situation in common. That is, they are compatible when there is at least one situation that is possible according to the two assertions". After having answered the compatibility questions, subjects answered an induction-problem question (see, Results and Discussion).

Results and Discussion. The truth-table task questions produced the expected high "possible" judgments of FF<not-A_not-C> and FT<not-A_not-C>; respectively 86.5% and 76.6%. This is but a mere replication of a well established truth-table task phenomenon (see, Schroyens, 2007a, in press a, for meta-analyses). The potential priming effect of these possibility judgments is obviously what interests us here. Given that about 80% of subjects explicitly judged <not-A_not-C> to be possible when assuming <if A then C> and <if A then not-C> to be true, and given that they were explicitly told what it means for two utterances to be compatible, one would expect these subjects to judge the two conditionals to be compatible. It seems but a simple application of a given rule, i.e., the definition of compatibility proffered by Handley et al. (2006). No such thing. There was no 'improvement' whatsoever. The conditionals were judged compatible by 21.98 % and 21.98% of participants answering respectively the first and second compatibility question. We take the complete failure of any subject to successfully apply Handley et al.'s (2006) definition as a suggestion that modal reasoners are not reasoning about the conditionals on the basis of possibilities and that the possibility-based compatibility is normatively inappropriate.

Of course, it has not been shown that suppositional-conditional theory cannot explain the incompatibility judgment of contrary conditionals. First, what has been shown is that if suppositional-conditional theory assumes a possibility-based judgment process and one thus makes a fair comparison by evenly assuming people make compatibility judgments on the basis of an overlap of the possibilities that are consistent with contrary conditionals, then it fails to explain the data just as much as mental-models theory does. Since suppositional-conditional theory is compatible with the notion of possibility-based judgments (cf. supra), one cannot engage in special pleading and presume without further justification that possibility-based judgments do not apply in the context of the novel compatibility-judgment task. Second, the results provide a clear indication that the notion of possibility-based compatibility judgments is best abandoned as a hypothesis about how people go about in judging that propositions are compatible. Though it is not an a-priori theoretical reason, the explanatory inadequacy of possibility-based compatibility judgments (both when using it in mental-models theory and suppositional-conditional theory) gives an empirical reason to abandon the idea that people spontaneously make possibility-based judgments in reasoning about their beliefs in the conditionals to judge whether they are compatible. It has been shown that they certainly have difficulty reasoning in line with the notion of possibility-based compatibility judgments. This concurs with Schroyens'
(2009, in press a) recent evidence that people are susceptible to the induction-problem and also have difficulty reasoning in line with possibility-based truth judgments. Indeed, mental-models theory provides an a-priori theoretical reason against the normative adequacy of possibility-based judgments. People will not tend to endorse conclusions that throw away semantic information, and this is exactly what one would do in endorse the conclusion that "if p then q" or "if p then not-q" is true when "not-q and not-q" is true.

To test people's sensitivity to the induction problem, the 141 participants in the present study also answered two induction-problem questions. (These problems were presented after the compatibility questionnaire, which means that they could not have affected the compatibility results that have been the focus of the present study and the preceding discussion). We asked them to "imagine a collection of 100 colored geometrical figures. You have identified the shape and color of 90 figures. Ten figures remain hidden and unidentified". An indeterminate problem-set described 65, 0, 20 and 5 TT, TF, FF and FT cases and explicated 10 figures were unidentified. A determinate problem-set described 50/15/20/5 different TT/TF/FF/FT cases and is determinate because there are TF cases falsifying the rule. (The presentation order of the determinate and indeterminate problem was counterbalanced). Participants were asked: "Is it possible to evaluate whether the rule is true or false, taking into consideration that there remain unidentified figures?". They selected one of three response alternatives: "No, for the time being, one cannot know whether the rule is irrefutably true or false", "Yes, the rule is irrefutably true" and "Yes, the rule is manifestly false". The determinate problem was correctly solved by 63.5% of people judge that the conditional rule was false about the incomplete set with 10 falsifying FT cases and the indeterminate problem was correctly solved by 79.7% of people judging that one cannot decide whether the rule is true, given a context where unidentified cases might still falsify the rule. These additional findings clearly indicate that people are susceptible to the induction problem (i.e, the loss of semantic information), and generally do not think a rule can be made true on the basis of cases that are merely consistent with this rule. This concurs with Johnson-Laird and Byrne's (2002) distinction between reasoning about possibilities on the basis of the presumed truth of the conditional and reasoning about truth on the basis of possibilities and further undermines the applicability of Handley et al.'s (2006) possibility-based notion of compatibility. A single case is insufficient to establish the truth of a proposition; hence this single case can a fortiori never be sufficient to establish that two propositions can be true at the same time.

Experiment 2 and 3: A Truth-based versus possibility-based notion of compatibility.

We have shown that when we use the possibility-based compatibility notion both mental-models theory and suppositional-conditional theory must predict that idealized reasoners should consider contrary conditionals compatible. This stands in stark contrast with the fact that modal reasoners do not consider them compatible. We suggest the theoretical gap between would (the descriptive) and should (the normative) is best filled by reconsidering the notion of compatibility, and this independent of whether one is partisan to one or the other theory. That is, it seems that people do not engage in reasoning about their beliefs to establish whether they are compatible but have a
different task construal and engage at least in part in reasoning from the truth of conditionals when they are asked to judge the conditionals are compatible.

We suggest that when people are considering the compatibility of their beliefs, modal reasoners are at least in part reasoning from their beliefs and adopt a higher-level interpretation of compatibility signifying two utterances can be true at the same time. It is such a notion, based on the assumption that people reason from the conditionals to judged their compatibility, that Handley et al. (2006) unevenly used to evaluate their own theory. Let us denote this truth-based compatibility (vs. possibility-based compatibility, defined by a shared true possibility). If <if A then C> is true, then TT<A_not-C> is false/impossible: it is never possible. This is not an issue of discussion (see, Handley et al., 2006, p. 563). However, if <if A then not-C> is true, then people think TT<A_not-C> must be possible. Since <A_not-C> cannot be possible and impossible at the same time, the conditionals cannot be true at the same time; hence they are truth-based incompatible.

The expert reader will note that it is not necessary TT<A_not-C> cases exist when uttering <if A then not-C>. The existential presuppositioniii that such TT cases are true possibilities (a.k.a. 'existential import' in predicate calculus) is a direct consequence of the mental-model theory's initialmodel principle, which is constrained by the truth-principle. Many critics have stated that according to the theory's truth-principle, people would represent true possibilities. This is only correct when one does not misinterprets "true possibilities" as if they are possibilities that make the conditional true. The theory's truth-principle states that "each mental model of a set of assertions represents a possibility given the truth of the assertions" (Johnson-Laird & Byrne, 2002, p. 653, italics added). That is, true possibilities are not a possibility that make the conditional true but are by definition cases that are possible given the conditional is true (and one is thus reasoning from the conditional). The initial-representation principle of the theory, which is based on the idea of cognitive economy, furthermore proffers that of all true possibilities people at first consider only TT. That is, when first being confronted with a conditional utterance like 'if A then C' people will pragmatically assume it is true (see Gilbert, 1991; Grice, 1975) and construct a representation of the "A and C" contingency. That is, assuming the conditional is true, TT is a true possibility: This is nothing more or less than the existential presupposition.

To investigate the existential presupposition we asked a first group of participants to construct a truth-table case distribution (Experiment 2). They indicate the number of cases of a particular type (TT, TF, ...) that would need to be present in a set such that it makes sense to assert a conditional rule about that set. If people make the existential presupposition, then they would spontaneously generate frequency distributions in which there are TT cases. Given the presumed meaning of a conditional, they would not include TF cases especially when it is made clear that "if the figure is circle, then it is colored red" does not say the same as "if the figure is a circle, then it is possible that the figure is red" (see, Schroyens et al., 2008, also see Bennett’s, 2003, "semantic occamism"). A second group of participants (Experiment 3) was given a description of the distribution of the different truth-table cases in a set and asked to evaluate whether it is possible/impossible to utter the conditional about the given set. If people make the existential presupposition, then they would tend to state <if A then C> is impossible when there are no TT<A_C> cases. When there are TF<A_C> cases, the rule would also be judged impossible. We therefore also gave participants the description of a set in which there
were neither <A,C> nor <A,not-C> cases. Hence, <if A then C> would not be judged impossible simply because there are falsying TF<not-A,C> cases.

Method. A total of 188 11th and 12th grade students at a Flemish high-school participated. Ninety-five students completed the production task (Experiment 2), whereas 93 students completed the evaluation task (Experiment 3). Figure 1 gives a reproduction of truth-table construction task about <if A then not-C> (translated from Dutch). The same content and format was used for <if A then C>, presented either first or second. After having completed the construction tasks, participants were asked to judge whether "if the figure is a circle, then it is colored red" and "if it is a circle then it is not colored red" are compatible or incompatible. Figure 2 gives a reproduction of the sentence-evaluation task. Evaluation-task participants (Experiment 3) were not given the compatibility question.

Figure 1. Reproduction of the truth-table construction task used in Experiment 2.

<table>
<thead>
<tr>
<th>Triangles that are coloured orange.</th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>Triangles that are not coloured orange. (e.g., yellow triangles)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non-triangles that are coloured orange. (e.g., orange squares)</td>
<td></td>
<td></td>
</tr>
<tr>
<td>Non-triangles that are not coloured orange. (e.g., green circles)</td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Imagine a set of 100 coloured geometrical figures. Take the following rule:

**If the figure is a triangle, then it is NOT coloured orange.**

Indicate for each type of coloured figure how many of such figures should be in the set such that it makes sense to make the above claim about the set. (Please note that the figure does not state: if the figure is a triangle, then it is possible that the figure is not coloured orange).

Figure 2. Reproduction of the sentence-evaluation task used in Experiment 3.

<table>
<thead>
<tr>
<th>Squares that are coloured blue</th>
<th>0</th>
<th>100</th>
</tr>
</thead>
<tbody>
<tr>
<td>Squares that are not coloured blue (e.g., yellow squares)</td>
<td>0</td>
<td>100</td>
</tr>
<tr>
<td>Non-squares that are coloured blue (e.g., blue rectangles)</td>
<td>50</td>
<td>100</td>
</tr>
<tr>
<td>Non-squares that are not coloured blue (e.g., green circles)</td>
<td>50</td>
<td>100</td>
</tr>
</tbody>
</table>

Is it possible to make the following claim about the set of coloured figures? (The rule does not need to give the best description; it just had to be possible without being contradicted).

**If the figure is a square, then it is coloured blue.**

A possible claim.

An impossible claim.

Is it possible to make the following claims about the set of coloured figures? (The rule does not need to give the best description; it just had to be possible without being contradicted).

**If the figure is a square, then it is NOT coloured blue.**

A possible claim.

An impossible claim.
Results and discussion. Some of the truth-table construction participants (N=30/95) had provided a likelihood or percentage instead of a frequency with the four truth-table cases. That is, the sum of cases they used did not match the total number of 100 figures in the set. We therefore dichotomized all scores. When either the frequency or likelihood was 0, it remained 0; otherwise participants had indicated that there had to be such truth-table cases (independent of however frequent or likely they thought this was). Table 2 presents the results.

First, for both rules, the vast majority of people (86.3% and 88.4%) used TT cases. These are TT<A_C> cases for <if A then C> and TT<A_not-C> cases for <if A then not-C>. That is, reasoners clearly tend to assume that there have to be "red circles" before one can make the claim that "if the figure is a circle, then it is red". This implies that when people use a truth-based notion of compatibility, two contrary utterances are incompatible. Second, Table 2 shows that the majority of people use falseantecedent cases for both rules. That is, some 'non-circular and non-red' figures are used and accepted in a set for which it both holds that "if the figure is a circle, then it is red" and "if it is a circle then it is not red". This implies that, if people were to use a possibility-based notion of compatibility, they would judge the two conditionals compatible. They clearly do not. There were 18 out of 95 participants (i.e., 18.9%) who judged the contrary conditionals compatible. This is a long way from the about 70% of participants who indicated that the two contrary conditionals share a common possibility. That is, as indicated by Experiment 1, participants do not seem to adopt a possibility-based notion of compatibility.

Table 2. Proportion of participants (N=95) who used the different truth-table cases to construct a set about which one can assert the conditional rule (Experiment 2).

<table>
<thead>
<tr>
<th>Rule</th>
<th>TT</th>
<th>TF</th>
<th>FT</th>
<th>FF</th>
</tr>
</thead>
<tbody>
<tr>
<td>If A then C</td>
<td>86.3</td>
<td>18.9</td>
<td>60.0</td>
<td>76.8</td>
</tr>
<tr>
<td>If A then not-C</td>
<td>88.4</td>
<td>14.7</td>
<td>70.5</td>
<td>77.9</td>
</tr>
</tbody>
</table>

In Experiment 3, the "evaluation-task" group evaluated both the <if A then C> and <if A then not-C> conditional about a set in which there were neither <A_C> nor <A_not-C> cases but some <not-A_C> and <not-A_not-C> cases. The if A then C rule was judged impossible by 86.1%. (The <if A then not-C> rule was judged impossible in 54.8% of cases, which is in line with the difficulty of processing negations: see, e.g., Schroyens et al, 2001). They could not have come to this conclusion because there were falsifying TF<A_not-C> cases in the set. The high 'impossible' rates therefore corroborates that people make the pragmatic implicature, i.e., existential presupposition that the TT<A_C> cases must be real possibilities before it makes sense to make a claim about the hypothetical relation between <A> and <C>.

General Discussion

Experiment 1, as well as Experiments 2 and 3, suggest that people are not reasoning about conditionals to decide whether they are compatible and, thus, do not use a possibility-based notion of compatibility. I suggested that instead they are reasoning from...
conditionals and use a truth-based notion in which two assertions are judged as compatible when they can be true at the same time. In mental-models theory \(<A \text{ and } C>\) is a possible case (and initially the only possible case considered) when reasoning from the assumption that \(<\text{if } A \text{ then } C>\) is true. At the same time, this same case is impossible when reasoning from the assumption that \(<\text{if } A \text{ then not-C}>\) is true. It follows that the two contrary conditionals are truth-based incompatible. They cannot be true at the same time. Experiment 2 confirmed the tendency for people to make the existential presupposition that there are (physically real or fictional) TT cases when the conditional is assumed true and these cases establish the falsity of the contrary conditional. In summary, first, mental-models and suppositional-conditional theory do not make opposing predictions and, second, the evidence suggests that people reason from the assumption of truth and adopt a truth-based notion of compatibility.

**Reasoning about the truth versus reasoning from the presumed truth of propositions**

31 How do the lower-level and higher-level definitions of compatibility relate? The former is defined as a function of possibilities, whereas the latter is defined as a function of truth. When we consider mental-models theory, these concepts are not psychologically equivalent:

32 "Discourse about the truth or falsity of propositions is at a higher level than mere descriptions of possibilities". (Johnson-Laird & Byrne, 2002, p. 653) Mental-models theory implies that when the conditional is assumed true (and people are thus reasoning from the conditionals), the false-antecedent cases are possible. It does not work the other way round: Truth goes down, but does not go up. The truth of a conditional implies that the false-antecedent cases are true possibilities, but the truth of a false-antecedent case does not imply that the conditional is true. This is a straightforward application of the semantic-information principle of mental-models theory. The semantic-information principle states that people will tend not to endorse conclusions whose semantic information is higher than the premise(s) it is based on (Johnson-Laird & Byrne, 2002, p. 652; also see Johnson-laird & Byrne, 1991, 1993). Semantic informativeness refers to the number of possibilities rule out by a proposition (but see, Dekeyser, Schroyens, Schaeken, Spitaels, & d’Ydewalle, 2000). The assertion "not-A and not-C" rules out 3 out of 4 possibilities; only the "not-A and not-C" combination is a true possibility. In contrast, the conditional rules out only 1 out of 4 possibilities. "All theories of the conditional agree that the only state of affairs that contradicts [if the cat is happy then she purs] 1 is a happy cat not purring (TF), and so all other cases are possible" (Evans, 2007, p. 54). That is, going from the truth of one truth-table case to the truth of the conditional proposition involves a loss of semantic information (3/4 vs. 1/4). It follows within mental-models theory that people will not tend to think that the conditional is true when a particular possibility is true. If follows, a fortiori, that people will not believe that not just one both two conditionals are true at the same time when they happen to share a common possibility.

33 Of course, the fact that the semantic-information principle is part of the mental-models theory (e.g., Johnson-Laird & Byrne, 2002) implies that Handley et al. (2006) is mistaken in its undefended special pleading and selective attribution of possibility-based compatibility to mental-models theory. It is true that until recently (Schroyens, 2009, in
press a; also see Barrouillet et al., 2008) many theorists never gave much attention to the semantic-information principle and the distinction between reasoning from versus reasoning about propositions. However, that is quite irrelevant. One should not critique a theory on the basis of the fact that proponents of that theory did not flesh out particular implications of the theory. When 'opponents' have not derived particular predictions, theoretical rigor and neutrality requires one still makes the best possible case to derive predictions from the 'opposing' theory (see the so-called principle of charity in Damer, 2003; also see Schroyens, 2007). Science advances more when one is able to form a valid critique of the stronger theory, as compared to an easy and superficial criticism against a weak straw-man version of a theory.

The distinction between reasoning from and reasoning about conditionals is also relevant to understand how suppositional-conditional theorists can claim that the subjective belief in "if p then q" is measured as the conditional probability of 'q, given p' while the theory at the same time can claim that TF<p.not-q> cases are impossible and show a conditional is false. If the TF<p.not-q> cases are truly impossible then their probability is zero. That is, P(p.not-q) = 0. It follows --given P(q|p)= 1 - P(not-q|p) -- that P(q|p) = 1. That is, how can suppositional-conditional theory explain that the subjective belief in the conditional is truly probabilistic and less than certain, when the theory also states that the interpretation of "if" can be non-probabilistic and people are categorically about TF being impossible? The solution to this conundrum is found in the distinction between reasoning from the assumption that the conditional is true versus reasoning about the truth of the conditional. When reasoning from the assumption that the conditional is true, they are invited to set their subjective belief in q, given p to unity: P(q|p) = 1. When people are reasoning about the conditional, it is actually this subjective belief in the conditional that needs to be determined. In this case people need to use their representation of what is possible and impossible to estimate it. As Ohm and Thompson (2008, p. 273) noted about conditional probabilities: "these probabilities, however, are not explanatory constructs. Rather they are mathematical summaries that represent the culmination of one or more underlying representational processes." In some way the suppositional-conditional theory needs to consider how people go about in reasoning on the basis of possibilities in order to establish their belief in the conditional. How exactly subjective probabilities are established, need not concern us here. The crux of the matter is that in doing so people are making possibility-based judgments in reasoning about the conditional on the basis of possibilities. It is quite something else when people are reasoning from their belief in the truth (or if not truth, then truthfulness) of conditionals. In this case, the conditional-probability theory makes the straightforward predictions that their belief in two contrary conditionals should be complementary. Indeed, P(q|p) = 1 - P(¬q|p).

General conclusion

As regards the compatibility of contrary conditionals, mental-models theory and suppositional theory are compatible – pun intended. When one makes a fair comparison, both mental-models theory and suppositional-conditional theory yield the same predictions from the same hypothesis about what 'compatible' means for modal reasoners. Though the negation of conditionals might still be a litmus test of suppositional conditional theory, the compatibility of contrary conditionals and the contrary beliefs expressed by these conditional is not. Handley et al.’s (2006) and our own
findings suggest people are in part reasoning from their beliefs and are not reasoning about their beliefs to evaluate whether they are compatible.

**BIBLIOGRAPHY**


NOTES

1. An anonymous reviewer has claimed that page 560 of Handley et al. (2006), from which I cite the possibility-based definition, does provide reasons for (in my view one-sidedly) considering possibility-based judgments within mental-models theory (but not suppositional-conditional theory). On page 560 one effectively finds an explication of how contrary conditional are compatible in mental-models theory, given one assumes possibility-based judgments. Assuming it is indeed the case that contrary conditional are incompatible according the suppositional-conditional theory (which I argue is not the case when evenly considering possibility-based judgments in this theory and is only the case when unevenly presuming a truth-based
compatibility judgment), this would indeed imply the theories do not agree (which is a stretch away from the claim that they “cannot agree” about the singular issue of compatibility judgments). But the trivial illustration/explication of possibility-based judgments in mental-models theory, is of course begging the question. Since when is an illustration of the consequences of a principle, that is, the application of a principle, a justification of applying this principle in the first place? Let me extrapolate. In the second world war millions of Jews (and other “deviants” such as homosexuals, political opponents, ...) were sent to the gas chambers and up until the sixties it happened in the US that black Americans were lynched because they were 'inferior': a rather simple application of the principle that they were considered inferior and not on equal footing such that their individual sovereignty was not to be respected. I fail to see that this implementation of an idea, justifies the principle.

There is a strong cross-theoretical consensus about the idea that many people often do not come to a conditional interpretation, but reason towards the so-called bi-conditional interpretation in which FT<not-A_C> is considered impossible (see Schroyens, 2007a, in press, for meta-analyses). When people have a bi-conditional interpretation of the contrary conditionals or even of just one of the two conditionals, they do not share a true possibility, and are thus not compatible. Handley et al. did not take this well-accepted fact into account in their argumentation, which is therefore weakened considerable already.

Mental-models theory claims that the antecedent of a conditional creates a context, and that the consequent is interpreted within that context. This context does not need to be real and does not need to correspond to a factual state of affairs. That is, the existential presupposition does not mean that TT cases are supposed to be real, factual states of affairs. One of the human marvels is indeed their imagination and ability to think hypothetically about other possible worlds.

ABSTRACTS

How do people go about in evaluating the consistency of their beliefs? Do people reason from or do they reason about their beliefs in judging whether they are compatible, or both? The paper investigates how people evaluate belief in contrary conditionals \(<\text{if } A \text{ then } C>\) and \(<\text{if } A \text{ then not-C}>\).

Experiment 1 (N=141) indicates people do not use a notion of possibility-based compatibility according to which claims are compatible when they share a common possibility: After being given this definition, there was no improvement of compatibility judgments even though more than 60% confirmed a shared possibility. Experiment 2 (N = 95) and Experiment 3 (N=93) test the alternative truth-based notion of compatibility, according to which contrary conditionals are incompatible because they cannot be true at the same time. The sets people construct for a true conditional invariably include “A and C” cases (Experiment 2) and “if A then C” is judged “un-assertable” about sets that do no include such cases (Experiment 3). Such “A and C” cases are at the same time impossible when “if A then not-C” is true. Findings thus suggest contrary conditionals are judged incompatible because they cannot be true at the same time.

Comment évalue-t-on la cohérence de nos croyances? Est-ce qu’on raisonne en prenant nos croyances pour vraies ou les remet-on en question afin de juger si elles sont compatibles? L’article présente une recherche sur l’évaluation des croyances en deux conditionnelles contraires: "si A alors C" et "si A alors non C". Expérience 1 (N = 141) montre que les gens n’utilisent pas une notion de compatibilité-des-possibilités, selon laquelle deux assertions sont
compatibles quand elles ont une possibilité partagée: Après avoir accepté cette définition, il n'y pas d'augmentation de jugements de compatibilité même si plus de 60% affirment une possibilité partagée. Les Expérience 2 (N = 95) et 3 (N = 93) testent la notion de compatibilité-de-vérité, selon laquelle des expressions sont incompatibles quand elles ne peuvent être vraies en même moment. Les ensembles de cas qu'elles composent pour un conditionnel vrai incluent les cas <A et C> (Expérience 2) et <si A alors C> est jugé "non-assertif" pour des ensembles qui n'incluent pas ce genre de cas. Les cases <A et C> sont en même temps impossible quand <si A alors ne pas C > est vrai. Les résultats confirment ainsi que les conditionnels contraires sont jugés incompatibles parce qu'ils ne peuvent pas être vrais en même temps.

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Keywords: compatibility, conditionals, opposite beliefs, reasoning, special pleading

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