Neoclassical General Equilibrium Theory as a Source of Powerful Concepts, Although Confronted with the Complexity of Market Economies

La théorie néoclassique de l'équilibre général, source de concepts puissants, mais confrontée à la complexité des économies de marché

Edmond Malinvaud
Edmond Malinvaud

Neoclassical General Equilibrium Theory as a Source of Powerful Concepts, Although Confronted with the Complexity of Market Economies

Avertissement
Le contenu de ce site relève de la législation française sur la propriété intellectuelle et est la propriété exclusive de l'éditeur.
Les œuvres figurant sur ce site peuvent être consultées et reproduites sur un support papier ou numérique sous réserve qu'elles soient strictement réservées à un usage soit personnel, soit scientifique ou pédagogique excluant toute exploitation commerciale. La reproduction devra obligatoirement mentionner l'éditeur, le nom de la revue, l'auteur et la référence du document.
Toute autre reproduction est interdite sauf accord préalable de l'éditeur, en dehors des cas prévus par la législation en vigueur en France.

Référence électronique

Éditeur : Association Œconomia
http://oeconomia.revues.org
http://www.revues.org

Ce document est le fac-similé de l'édition papier.
© Association Œconomia
Neoclassical General Equilibrium Theory as a Source of Powerful Concepts, Although Confronted with the Complexity of Market Economies

Edmond Malinvaud*

General equilibrium theory is an instrument for the analysis of market economies. The wide range of concepts used in general equilibrium stems from the complexity of market economies. In this article, I provide an overview of some important steps in the development of general equilibrium theory. Then, I show that theoreticians working with general equilibrium models will seldom arrive at definite conclusions without precise references to quantitative observations.

Keywords: general equilibrium, Debreu, elasticity of substitution, Cowles Commission, market economies

La théorie néoclassique de l’équilibre général, source de concepts puissants, mais confrontée à la complexité des économies de marché

La théorie de l’équilibre général est un instrument d’analyse des économies de marché. La grande variété de concepts utilisés en théorie de l’équilibre général découle de la complexité des économies de marché. Dans cet article, je propose un aperçu de quelques étapes importantes du développement de la théorie de l’équilibre général. Je montre ensuite que les théoriciens qui utilisent des modèles d’équilibre général parviendront rarement à des conclusions sans référence à des observations quantifiées.

Mots clé: équilibre général, Debreu, élasticité de substitution, Cowles Commission, économies de marché

JEL: B2, D5

* CREST and Collège de France

The material of this article was presented at the Paris 2007 conference “General Equilibrium as Knowledge”. The author wishes to acknowledge the remarks by two anonymous referees. The editors thank Maxime Desmarais-Tremblay for technical assistance in the preparation of this article.
Three sentences will summarize the simple message of this article. First, the theory of general equilibrium was built in order to provide not a justification of capitalism but rather an instrument for the analysis of market economies. Second, indeed neoclassical general equilibrium theory is an essential part of our discipline because it provides a wide range of concepts, which are all essential for our reflections about such economies. Third, the multiplicity of concepts follows from the complexity of these economies, which is also responsible for the complexity of the theory. Hence, applications of the theory are challenging: definite conclusions would seldom emerge without precise reference to quantitative observations.

My talk will have four parts. The first one will be focused on my maturing as an economist during the forties and fifties. The second one will be on Gérard Debreu’s contributions to general equilibrium theory (it seems to me that this particular selection provides an excellent observatory for the achievements reached by the axiomatic, purely mathematical, approach to the theory in question). In the third part I shall consider the more or less marginal topics on which I focused my attention within the same research program. The theme of the fourth part will be an illustration of the type of research that may put order in the more empirically oriented applications of equilibrium theory.

1. The Allais seminar and the Cowles Commission

In my youth in the 1930s in the city of Limoges, I became familiar with industrial competition from abroad, unemployment, the struggles emerging from the working class and the presence of poverty. As I reached the age of 18 in 1941, awareness of such facts clashed with my discovery of the economic discipline, to which I was somewhat exposed in my occasional study of law, on the side of a main study of mathematics and sciences. Up to 1947, as the war was going on, I read a good deal of economics in such books as the Histoire des doctrines économiques by Gide and Rist (1944) or those dealing with business cycle facts or still with a wide range of theories. Although I had begun to also take courses in economics, my readings were very eclectic and my understanding similarly so.

Meeting Maurice Allais at my age of 24 was a first great chance, because he immediately provided me with what was necessary to put order in my knowledge. He had just published two books in French on economic theory (Allais, 1943; 1947), which he had worked out himself in the relative isolation of the war. He was then starting to organize a seminar where young students would be working with him and helping him to be aware of the recent literature on economic theory. We were soon organized as a team of fewer than ten active
members, inspired by the same research spirit as was found among young mathematicians, or other scientists in France at the same time. I shall just mention the two close to whom I would most work during the following years: Marcel Boiteux and Gérard Debreu.

My second great chance was that Allais obtained for me a Rockefeller grant to work at the Cowles Commission in Chicago. From June 1950 to July 1951 I would share the life of an excellent international research group, which had been moreover revitalized in 1943, in order to work on a number of issues belonging in particular to the methodology of econometric inference and to general equilibrium theory. My visit was rather short, but it served to build tight links with many others, including by the way John Chipman as well as Gérard Debreu. We would often meet later in Europe or in the US. My duties in France would even permit me to accept two visiting professorships in Berkeley, for seven months each, one in 1961, the other in 1967.

2. Gérard Debreu’s contributions

The neoclassical general equilibrium theory had developed since Léon Walras’ book (1874). An important addition to it was brought by Vilfredo Pareto, showing in 1909 that, in some sense, a competitive equilibrium was efficient, i.e. “Pareto efficient” as it was later labeled. Work on the theory in question was even particularly active during the 1930s and 1940s with such books as Hicks (1939), *Value and Capital*, Allais (1943), Samuelson (1947), *Foundations of Economic Analysis*, all tending to the emergence of a standard formalization with supplies and demands of producers and consumers. But, in one respect the mathematical systems that had been worked out were lacking in the rigor that pure mathematicians were then requiring: economists had stopped at checking that the number of variables to be explained was equal to the number of independent equations, which was notoriously insufficient to prove existence of an equilibrium. A few mathematicians meeting in Vienna, such as Wald and von Neumann, had explored ways to bypass the difficulty for quite particular models. Actually, it was the beginning of a period in which mathematical rigor was going to penetrate the theory of general equilibrium. As a graduate from Ecole Normale Supérieure, Gérard Debreu then realized such was going to be his job. I shall record the results in four points.

Certainly, Debreu was not the only one to endorse that program. I could mention a list of others. But he was so typical and I knew him so well that it is convenient for this part of my presentation to restrict my attention to him. He contributed to many fundamental issues in general equilibrium theory: existence of equilibrium, robustness of
the competitive equilibrium and specificity of excess demands. The unity of his project was to build a mathematics of economies.

The problem was to exhibit a set of fairly general sufficient hypotheses, which would imply this existence. The first result in this respect was published in 1954 in *Econometrica* jointly by two authors (Arrow and Debreu, 1954), with the title “Existence of an equilibrium for a competitive economy”. The selection of the hypotheses was so persuasive that the phrase “Arrow-Debreu economy” was for long familiar. But Debreu was not fully satisfied: he was concerned with finding a better set of sufficient conditions and a more direct proof of existence of an equilibrium. He kept investigating the issue further with results contained in his 1959 book (*Theory of value: An axiomatic analysis of economic equilibrium*), and later complemented in the article “New concepts and techniques for equilibrium analysis” published in 1962 in the *International Economic Review*.

For economists who know the complexities of market structures it is natural to wonder about the relevance of an equilibrium concept embodying the perfect competition hypothesis. Indeed, a substantial part of economic theory is validly devoted to imperfect competition. But intuition also suggests that, when the relevant markets are all large, the competitive general equilibrium may provide a sufficiently good approximation. Can that intuition be supported by a rigorous argument? Already in 1881 the Irish economist Edgeworth provided such an argument for a simple case, in which he showed that a market would tend to be competitive if the number of participants would increase indefinitely, each one tending to become negligible.

This result was greatly generalized in 1963 by Debreu and Scarf proving “A limit theorem on the core of an economy”. The concept of core was there borrowed from the theory of games, which was considered as making sense under imperfect competition. One could then speak of the robustness of the concept of a competitive equilibrium. All the more so as it turned out that similar limit theorems also applied to almost all concepts which could be substituted to that of core in order to characterize the outcome of imperfect competition (Aumann, 1987).

How specific are the results of the classical theory of general equilibrium? In the late 1970s many students of general equilibrium theory were disappointed to learn that, taken alone, the theory in question was imposing only very few restrictions on the set of aggregate demand and supply functions of the various goods and services. The system to be solved in order to determine the values of all endogenous variables in the case of $n$ goods or services was boiling down to $n$ equations, each one stating that the net aggregate demand of a particular good or service was equal to zero at an equilibrium. Under the system of hypotheses that was most commonly posed,
each net aggregate demand function had to be continuous and homogeneous in terms of prices. Moreover, the set of the $n$ aggregate net demand functions had to satisfy the “Walras law”, stating that the global value of the sum of all such net demands had to be identically equal to zero. But, as soon as these properties held, any specification was a valid case according to the theory. Debreu published a general proof of this result in 1974 ("Excess demand functions", *Journal of Mathematical Economics*, vol.1).

Surprisingly, this mathematical result was taken as destructive by many economists, who were inclined to then argue that not only was the theory false, for such or such reason, but it was also void. This judgment clearly made no sense. The result meant only that, for meaningful applications of the theory, extra information besides perfect competition was also needed. The attitude in question was really revealing ignorance of the mathematical discipline and of the role of empirical information in applications of theories.

From the outset, Debreu’s project was to build a mathematics of economies, which had to be rigorous and couched down in the concise language that was his distinctive mark. This lead him to define the concept of a measure space of economic agents, which allowed making sense of the idea that economic agents would be “neighboring”. More generally Debreu wanted to explore meaningful categories such as “regular differentiable economies”, which were appropriate for the study of a number of issues emerging in applied fields (e.g. his article in the 1976 *American Economic Review Proceedings*).

### 3. Approaches of mine, here and there

When I left the Cowles Commission in the summer of 1951 I had a definite research project in mind, which had born there and would soon be brought to completion. Similarly, part of my time during my two visits in Berkeley was devoted to the search for particular Pareto efficient solutions to the planning problem. Research about ideal exchanges of risks on competitive market later led to the publication of two theoretical articles (Malinvaud, 1972; 1973). I will first present my work on capital accumulation and efficient allocation of resources (Malinvaud, 1953) and planning (Malinvaud, 1967). I then turn to the question of risk allocation and to elasticities of substitution.

Economists working at Cowles in 1950-51 were well aware that general competitive equilibria could be applied to economies where markets for intertemporal trades were assumed to exist. In that case two goods with the same physical nature had to be considered as
distinct “commodities” as soon they were meant to be available at different times. However, the number of commodities had to be infinite if the theory had to cover cases in which the time horizon was not assigned a given bound. Moreover, some of those economists who had been working on capital theory had particularly brought their attention to “stationary states”, those in which the same technologies and needs, the same productions and consumptions, were assumed to occur period after period.

I was then perceiving a double challenge: to explicitly extend the classical theory of the competitive general equilibrium to the case of an infinite horizon, and to show how stationary equilibria could appear as particular cases of general equilibria. That was actually a mathematical challenge in connection with two distinct branches of then modern mathematics. In particular an additional hypothesis was needed in order to exhibit a price system that was in conformity with those defined for economies with a bounded horizon. So, I describe my article as having aimed at consolidating the link between the general equilibrium theory of the allocation of resources and capital theory.

More than most of my colleagues I was involved with the planning problem in market economies. My first published article, in 1950, had the title “L’expérience travailliste et la pensée économique anglaise”. It was a survey of publications in England, in the immediate postwar years, about whether and how to plan the economy. The focus was notably on the book published by Abba Lerner in 1944 with the title The Economics of Control. Principles of Welfare Economics. My interest in the planning issues lasted throughout two decades, maintained in particular by my operational proximity to the Commissariat Général du Plan and my intellectual proximity to Pierre Massé.

There already was a long tradition in economics about economic planning with its two aspects: (1) management of public utilities, since the time of Jules Dupuit (1844), (2) more generally the welfare economics of future trends in prices and productions. In the 1960s in France and a few other market economies, a standing problem in this respect was to know how to best organize the exchange of information where the economy was operated with a high degree of decentralization. I wrote then several articles on this subject; the best known probably being “Decentralized procedures for planning” (Malinvaud, 1967).

A natural extension of the theory of general equilibrium beyond the intertemporal aspect was contemplated in the direction of risk taking. That was actually a multidimensional challenge, which greatly outgrew my own contribution. The latter aimed at investigating the impact of a remark made in particular by Kenneth Arrow. Many risks are individual in a modern economy, in the sense that each
such risk bears on a person or a firm, which may be hit or not with a given probability. But many persons are exposed to similar risks with the same probabilities. These individual risks may then be taken as interchangeable by insurance companies. Given the laws of large numbers and perfect competition, the value of the insurance premium would then follow. This proposition was examined, at two distinct levels of rigor, in my “The allocation of individual risks in large markets” (Malinvaud, 1972), and “Markets for an exchange economy with individual risks” (Malinvaud, 1973).

I consider these results as informative to some extent. But discussing the allocation of risks has other dimensions, which were early identified and follow from the fact that it is unrealistic, in market economies exposed to risks, to neglect the fact that information of agents are imperfect and known to be imperfect in different ways. So, a completely different approach had to be followed by younger economists.

It would be a mistake to consider today neoclassical general equilibrium analysis as an outdated branch of economic theory. Actually this branch still serves for meaningful developments (see for instance Samuelson (2004)). As an illustration of that, I will present one branch of analysis that is also now frequently discussed: aggregate elasticities of substitution between factors of production. Those characteristics of equilibrium matter in two long standing economic theories dealing respectively with growth and income distribution. Now they may turn out to greatly matter also in the prospective analysis of the environment, where risk of overuse of natural resources may be the focus of an increasing concern. It so happens that I recently used a simple general equilibrium model in order to investigate the determinants of these aggregate elasticities (Malinvaud, 2006). Here is a brief presentation of the model and of some of its aggregate implications.

4. A general equilibrium model

4.1. The model

There are \( n \) industries. Each industry \( h \) (\( h = 1, 2...n \)) produces output \( y_h \) of good \( h \) from inputs \( x^h_i \) of \( m \) factors (\( i = 1, 2...m \)) with a constant-returns-to-scale technology. Production functions \( f^h \) are twice differentiable. Markets for goods and factors are assumed perfectly competitive. At equilibrium \( p_i \) is the price of factor \( i \) and \( q_h \) that of good \( h \).

The system of demand functions for goods is directly posed at the aggregate level and written as:

\[
\ln y_h = g^h (\ln q_1, \ln q_2, ... \ln q_n; \ln r), h = 1, 2 ... n
\]  

(1)
with income $r$ given by:

$$r = \sum_{h=1}^{n} q_h y_h$$

(2)

The functions $g^h$ are differentiable, $g^h_k$ being the derivative with respect to $\ln q_k$ and $g^h_r$ the derivative with respect to $\ln r$. In what follows the $n$-dimensional square matrix $C$ is defined as having elements $c_{hk} = -g^h_k$.

Market shares and cost shares in a reference equilibrium will be denoted by $v_h$ and $w^h_i$:

$$v_h = q_h y_h / r$$

$$w^h_i = p_i x^h_i / q_h y_h$$

(3a)

(3b)

with the average cost shares:

$$\bar{w}_i = \sum_{h=1}^{n} v_h w^h_i$$

(4)

and we shall use the notation:

$$z^h_i = w^h_i - \bar{w}_i$$

(5)

The supply of factor $i$ in the economy will be taken as exogenous, denoted by $x^s_i$. Equilibrium will require:

$$\sum_{h=1}^{n} x^h_i = x^s_i$$

(6)

The purpose is a comparative static analysis of the infinitesimal changes such as $dx^h_i$ or $dp_i$ affecting factor inputs $x^h_i$ or factor prices $p_i$. We focus here on relative changes, which we denote by capital letters:

$$X^h_i = \frac{dx^h_i}{x^h_i}$$

(7a)

$$P_i = \frac{dp_i}{p_i}$$

(7b)
The outcome of the analysis is the set of relations linking relative changes in the endogenous variables to possible relative changes $X_i^S$ in factor supplies. The aggregate elasticities of substitution are given by the system of the $m$ equations connecting the relative changes in factor prices $P_i$ to the relative changes in factor supplies $X_i^S$. It turns out that this system is conveniently written in matrix form as:

$$X^S = D^{-1} \bar{w} \left[ A^S - ZD_pC^tZ \right] P$$

(8)

Our concern will now be to know the implications of this system, which must be first read and explained.

### 4.2. Some implications of the model: the two channels

The two column-matrices (i.e. vectors) (of relative changes) $X^S$ and $P$ have $m$ components (the number of factors). The other matrices are characteristics of the reference equilibrium, in comparison to which changes are measured. There are six such matrices: $C$ was defined above and characterizes the price elasticities in the system of the demand functions for goods; $\bar{Z}$ is an $m \times n$ matrix with elements given by (5) and $^t\bar{Z}$ its transposed: they characterize the dispersion of cost shares across industries; $D_v$ is the diagonal matrix defined by the $n$ industry market shares given by (3a), $\bar{D}_w$ is similarly the diagonal matrix defined by the average cost shares defined by (4). The matrix $A^S$ deserves a particular attention and plays a crucial role. It is defined by the weighted average:

$$A^S = \sum_h v_h A^h$$

(9)

of matrices $A^h$, each one being a function of the first and second order elasticities of factor utilizations in its industry $h$. For simplicity the formula leading to the computation of $A^h$ will not be given here. Let me simply say that it defines a kind of pseudo-inverse of a matrix $B^h$ involving first and second order derivatives of the production function $f^h$.

Actually, the form of the square bracket in the right-hand side of (8) illustrates the idea, often expressed in the literature, according to which there are two channels of adaptation of factor prices to changes in factor supplies. The first channel involves in each industry the direct substitution of relatively abundant factors to factors becoming scarcer (hence the role of matrices $A^h$). The second channel is indirect and reflects two other adaptations: first, the relative prices of goods change; the price increases for goods that use much of the scarcer
factors and decreases for those that use little of them; second, this change in prices induces substitutions between demands for goods. The importance of this second channel is reflected in the second term of the square bracket of formula (8), matrix $Z$ giving the importance of changes in prices of goods, and matrix $C$ that of substitutions between demands for different goods.

From recognition of the existence of those two channels, inference was often drawn that their effects would add up and therefore elasticities of substitution would be larger at the aggregate level than on average at the industry level. But this intuition failed to be always right, far from that. The implications to be drawn from equation (8) are actually complex, as the equation itself is not transparent.

The most easy case occurs when the model is written for two factors only, for CES production functions with all the same elasticity of substitution $\sigma^P$ (log-linearity would follow from $\sigma^P = 1$) and for a homothetic CES system of demands for goods ($\sigma^C$ being its elasticity of substitution). In that case (8) boils down to:

$$X_2^S - X_1^S = - \left[ \sigma^P + \left( \sigma^C - \sigma^P \right) \frac{\text{Var}(w)}{\bar{w}_1 \bar{w}_2} \right] [P_1 - P_2] \quad (10)$$

with

$$\text{Var}(w) = \sum_h v_h (w^h_1 - \bar{w}_1)^2 \quad (11)$$

(the same value of the variance holds when $w_1$ is replaced by $w_2$ in the formula). The long square bracket gives the aggregate elasticity of substitution, $\sigma^S$ say. It is equal to the industry elasticity $\sigma^P$ if and only if $w^h_1 = \bar{w}_1$ for all $h$, or $\sigma^C = \sigma^P$. Except for that, $\sigma^S > \sigma^P$ if and only if $\sigma^C > \sigma^P$. The conjecture that $\sigma^C > \sigma^P$ failed as soon as the elasticity of substitution between the demands for goods is smaller than the industry elasticity of substitution. Attached to $\sigma^P$ the multiplier $\frac{\text{Var}(w)}{\bar{w}_1 \bar{w}_2}$ (a measure for the heterogeneity of cost shares between industries) exhibits the fact that aggregation attenuates the factor substitutability present within industries.

The case in which formula (10) applies is very special. My article examines at some length many respects in which reality is more complex, so that the results derived from equation (8) are not as easily characterized.\(^1\) It would, of course, be out of place to record here these results of a research taken as an example.

---

\(^1\) What are the implications of the following changes with respect to the specification leading to (10)? - if the number of factors is larger than 2, - if the elasticities of substitutions $\sigma^h$ of the industry CES production functions differ, - if the production functions are more complex than CES, such as nested — CES, - if the income elasticities of the demands for goods differ from 1 and from one another.
The academic literature in economics now contains other similar applications of general equilibrium analysis. It seems to me that they should not be neglected in the history of economic theory because they may reveal a valuable evolution beyond the building period up to the end of 1970s.

References


Malinvaud, Edmond. 2006. Aggregate substitutabilities between factor demands. CREST. working paper.


