Philosophia Scientiæ

Travaux d'histoire et de philosophie des sciences

21-1 | 2017

Homage to Galileo Galilei 1564-2014

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Electronic version

URL: <http://journals.openedition.org/philosophiascientiae/1238>

DOI: 10.4000/philosophiascientiae.1238

ISSN: 1775-4283

Publisher

Éditions Kimé

Printed version

Date of publication: 15 February 2017

Number of pages: 35-54

ISBN: 978-2-84174-801-3

ISSN: 1281-2463

Electronic reference

Antonino Drago, "What Was the Role of Galileo in the Century-Long Birth of Modern Science? ", *Philosophia Scientiæ* [Online], 21-1 | 2017, Online since 15 February 2019, connection on 30 March 2021. URL: <http://journals.openedition.org/philosophiascientiae/1238> ; DOI: <https://doi.org/10.4000/philosophiascientiae.1238>

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What Was the Role of Galileo in the Century-Long Birth of Modern Science?

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Résumé : Quel est le rôle de Galilée dans la naissance séculaire de la science moderne ? Je réponds à la question ci-dessus à la lumière de deux nouveaux éléments. Dès le xvi^e siècle, Nicolas de Cues, quoiqu'il ne pratiquât pas la science expérimentale, a anticipé une partie substantielle de la révolution copernicienne et de la naissance de la méthodologie galiléenne. Le deuxième élément est l'introduction d'une nouvelle conception des fondements de la science ; ils sont définis comme constitués de trois dialectiques. Après cette définition, la naissance de la science moderne correspond à un très long processus historique qui se conclut à notre époque. Sur cette longue période, Galilée ne fut pas seulement le premier à recourir à une méthodologie scientifique, mais aussi presque le seul à jamais avoir été conscient de l'ampleur des enjeux intellectuels de l'entreprise scientifique.

Abstract: I answer the above question in the light of two new elements. Firstly, in the 15th century while Cusanus did not practice experimental science, he substantially anticipated both the Copernican revolution and the birth of Galilei's methodology. Secondly, I shall introduce a new conception of the foundations of modern science which are constituted by three dialectics. In this light, the complete birth of modern science, whose scope was so broad, required a very long historical process, which was completed in recent years. Within this long time span, Galileo was not only the first to practice a scientific methodology, but also almost the only scientist ever to be aware of the intellectual breadth of scientific enterprise.

1 Introduction: What are the foundations of science? The three dialectics

The role played by Galileo in the birth of modern science depends on two questions: What is the definition of modern science, in particular its foundations? How long was the process of birth of modern science? Without doubt, the foundations of science are constituted by the well-known dialectic¹ between experimental data and mathematical hypothesis. But after that, what in precise terms?

I suggest the following definition of the foundations of science that I obtained on the basis of four decades of historical works on the main theories of logic, mathematics and physics; they are constituted by two more dialectics of a theoretical nature; one between the two kinds of infinity (either potential, or actual) and a dialectic between the two types of organization of a theory (either deductive form principles-axioms, as Aristotle theorized in ancient times, AO, or aimed at the solution of a crucial problem, PO). The infinity dialectic was formalized, by, on one hand, classical mathematics relying on the actual infinity (AI) (e.g., Zermelo's axiom) and on the other hand by constructive mathematics relying on (almost) only potential infinity (PI) [Markov 1962], [Bishop 1967]. The organization dialectic was formalized by means of the kind of logic managing it; on the one hand classical logic, managing AO (e.g., in Euclid's *Elements*) and, on the other hand, non-classical logic, in particular intuitionist logic [Dummett 1977] managing PO [Drago 1990, 2012b]. When a scientist builds a theory, each theoretical dialectic appears under its formal aspect; a formal alternative is opposite to and incompatible with the other alternative; the scientist has to choose one. In sum, both dialectics constitute two scientific dichotomies. Once the choices are made, each dialectic is dissolved and the scientist proceeds within the scientific realm to establish the notions and principles of his theory. However, when we consider all the theories of a scientific discipline—e.g., physics—we again recognize these theoretical dialectics in the different couples of choices on which the various theories rely. In the light of the above illustrated foundations of science, the question of the birth of modern science receives a first, partial answer: *modern science was born when not only its method for producing accurate results was recognized, but also its foundations*, i.e., the two theoretical dialectics. Since the alternative choices of these dialectics were not formalized and recognized as relevant for science before the second half of the last century, we obtain a surprising result: *the birth of modern science is a historical process spanning*

1. I use the word “dialectic” in the intuitive sense, but in a Platonic sense as both authors [Hopkins 1988], [Counet 2005] do. It is not the Hegelian dialectic (that transcends reality through a dynamic of the Absolute Spirit); roughly speaking, it is given by two polarities that are not necessarily on the same plane. It may be resolved by either a reconciliation (as in the first dialectic) or by a choice between the two alternatives of the dialectic (as in the cases we will see in the following).

several centuries, at least half a millennium. In the following sections, we will detail this long process in relation to Galileo. Notice that the three dialectics of the foundations of science are mutually independent. Thus, according to the above definition, *the question about the birth of modern science has to be disentangled into the three questions about when each of these dialectics emerged and was then established, at first in intuitive and then in formal terms.*

In what follows, I will deal with them by considering each dialectic at a time.

2 The dialectic experiment/ mathematical hypothesis. Its historical birth and its problematic definition

Certainly, the historical start of modern science was marked above all by the birth of the experimental method through its innumerable applications giving new scientific results. Even a first, rough definition of this method was enough to mark an unprecedented novelty with respect to the philosophical world of the ancient Greeks. Rightly, historians of science attributed a crucial importance to this method. However, regarding this dialectic it is no longer possible to assume a positivist viewpoint, which attributes an eminent role to Galileo because he, more than any other, set hard experimental facts against the idealistic philosophical tenets of Aristotelian philosophers. If the experimental method was what positivist historians claim, that is, dealing above all with hard, experimental facts, it would be easy to determine a date for the births of both scientific activity and the method governing it. Instead, Grosseteste, Francis Bacon, Cusanus, Lavoisier, etc., suggested different aspects of this dialectic. Moreover, Galileo is a complex figure; in contradiction to the above appraisal, he also declared that he could obtain scientific results without experiments [Galilei 1638, II Day]. In addition, the question of whether Galileo's activity conformed to a well-defined experimental method is largely undecided. For instance, about Galileo's capital achievement—the discovery of the law of the accelerated motion—Galluzzi wrote:

[...] it is not still made irrefutably clear which element of the Galilean investigation was decisive in such an enterprise: either the natural deduction or the observation, either the geometrico-mathematical analogy or the experiment. [Galluzzi 1979, 158]

Notwithstanding having pondered for three centuries on this dialectic, philosophers of science did not suggest any common agreement on its main features. We know that not only positivist, but also neo-positivist philosophy, although supported by excellent scholars, failed in this task. No surprise if this unsatisfactory situation is presented by Feyerabend as no method at all in science

[Feyerabend 1975]. Recently, two scholars wrote an even more discouraging appraisal:

[...] More recent debate has questioned whether there is anything like a fixed toolkit of methods which is common across science and only science. [Andersen & Hepburn 2015]

The plain conclusion to be derived from these unsuccesses is that this dialectic is philosophy-laden. Indeed, both Koyré and Lindberg stressed that the establishment of the experimental method was the result and at the same time the cause of a profound change not only in the methodology aimed at obtaining answers from nature, but also in the metaphysical conception of reality [Koyré 1957], [Lindberg 1992].

3 The dialectic experiment/ theoretical hypothesis. Its historical birth and Cusanus' contributions

In such a context of uncertainty about this dialectic, new results concerning how Cusanus suggested combining experimental data with mathematical hypotheses are important. This relationship of combination was essentially new with respect to Greek philosophy, which considered a conjunction between mathematics and reality to be impossible. It was new also according to the idea that was staunchly emphasized by Koyré, i.e., modern science overcame the finiteness of the Greek world (Aristotle) by introducing a mathematics explicitly involving infinity, i.e., either potential infinity (i.e., unlimited), or actual infinity. According to some scholars this dialectic started, although in a reduced form of a mere programme, in the 13th century with Robert Grosseteste's (1175-1253) book *De Luce* [Crombie 1994, II, 319], [Lewis 2007]. However, subsequently, in the 15th century Cusanus (1401-1484) suggested important aspects of this method, although he intermixed them with theological, philosophical and cosmological subjects, all treated in a somewhat obscure way [Cusanus 1972]. First, he provided an accurate philosophical basis. According to him the two words *mens* and *mensura* share the same root; the *mens* connects itself to reality through a *mensura*; the *mens* mutually compares through proportions the numbers obtained (remember that a proportion was the only mathematical technique used in physics before the time of Galileo and Descartes) and moreover it mutually combines the corresponding concepts in order to obtain theoretical constructs.²

2. The "Introduction" by Graziella Federici Vescovini to [Cusanus 1450] is very informative about current scientific knowledge in Cusanus' time. Apart from her—in my opinion too severe—criticisms, she cleverly summarizes Cusanus' conception of human knowledge: "The mind is thus specular simplicity that all obtains from its capability of measuring, numbering and representing", [Federici Vescovini 2003, xxx].

In addition, in the book *De staticis experimentis*³ Cusanus indicated the main features of an experimental science through the measurements of a particular physical magnitude, weight:

And thus, by means of experiments done with weight-scales he [the physician] would draw nearer, through a more precise surmise, unto all that is known. [Cusanus 1450, 608, 164]
[...] Experimental knowledge requires extensive written records. For the more written records there are, the more infallibly we can arrive, based on experiments, at the art elicited from the experiments. [Cusanus 1450, 615, 178]

Notice that in ancient times weight, being considered a quality of a body, was extraneous to geometry. Its quantification as a quantity decisively introduced scholars to the quantifying of a multitude of other physical properties [Crombie 1994, 423].⁴ Furthermore, in the history of physics the transition from the theoretical concept of absolute weight to the concept of relative weight played an exceptional role. According to Koyré, Galileo considered only absolute weight, because he thought there was one single centre of the universe, i.e., the Sun, from which he could not free himself [Koyré 1978, 25 ff.]. Before him, Cusanus had already abolished any centre of the universe whatsoever as well as any fixed locations for celestial bodies.⁵ In fact, in the above mentioned book Cusanus used not so much weight but weight difference. Remarkably, by means of a Roman scale together with a hourglass Cusanus wanted to determine other physical quantities: e.g., the volume of a body, its specific weight, the weather, the temperature, the sound, the magnetic force, the weight of the air by means of the so-called leaning tower of Pisa experiment, etc.; so that he verbally introduced the indirect measures and, ultimately, the doubly artificial apparatus of measurement. One historian evaluates Cusanus' work in the following terms:

3. In the following, I will refer to Jasper Hopkins' English translations of almost all Cusanus' online books <http://www.jasper-hopkins.info/>.

4. About this point, Koyré, strangely enough ignored Cusanus' suggestions which constituted those advances that Koyré himself considered a crucial step in the history of the scientific method: "It is ironic that two thousands years beforehand, Pythagoras had proclaimed that number is the same essence of things, and the Bible had taught that God had founded the World on "number, weight, measure" [Wisdom, 11, 21]. All people reiterated that, nobody believed that. Surely, nobody before Galileo took that seriously. No one attempted to determine these numbers, these weights and these measures. No one attempted to count, to weigh, to measure. Or, more exactly, no one had the idea of counting, of weighing and of measuring. Or, more exactly, no one ever sought to get beyond the practical uses of number, weight, measure in the imprecision of everyday life. [...]" [Koyré 1961, 360]. It is apparent that Cusanus did exactly what Koyré stressed as lacking and he did it inspired by the same Biblical passage.

5. See Santinello for innovations and advances of Cusanus in science [Santinello 1987, 105–109]. In particular, notice that Copernicus knew Cusanus' modern view of Heaven without any center [Klibansky 1953].

In the mid-fifteenth century appeared what should have been the crowning work of this genre, the *Idiota de staticis experimentis*, from the pen of Cardinal Nicholas of Cusa. This incidental piece from one of the best-known philosophers and churchmen of the time does not appear to have attracted much attention. It advocates the use of balance and the comparison of weights for the solution of a wide range of phenomena, ranging from mechanics to medicine, and including those of chemical. He advocates differences in weights as a guide to the evaluation of natural waters, the condition of blood and urine in sickness and health, the evaluation of the efficacy of drugs and the identification of metals and alloys. It would be difficult to find a more specific prescription for what scientists were actually doing two centuries later. [Multhauf 1978, 386]

A century and a half after Cusanus, the first of Galileo's three periods of activity, was devoted especially to the study of physical phenomena relating to weight (e.g., a rolling ball on an incline, a pendulum) [Wisn 1974, 136–161]. This is the physics of the Earth, which, unlike the celestial one, which is based only on observation, is manipulative, as is modern science. Later, Newton's use of the new, marvellous mathematics of infinitesimal analysis assigned instead the highest role to celestial physics, despite the fact that the foundations of this mathematics were unknown and manifestly linked to the metaphysics of actual infinity [Newton 1687]. Yet, three centuries after Cusanus and one century after Newton, chemistry was born. This event was a "postponed revolution" (a chapter title of [Butterfield 1949]), since, being influenced by his idealistic mathematics, Newton has led chemists in a misleading direction, relying on several metaphysical notions: absolute space, absolute time, fixed and perfectly hard atoms, gravitational force as constituting the intermolecular links. Lavoisier freed chemistry from all of a priori notions by appropriately re-defining the experimental method for his field of research, i.e., all that concerned the complete methodology of chemical reactions. Lavoisier founded chemistry by means of only those physical measurements which had been suggested by Cusanus, i.e., weight differences that he considered between the reagent substances and the compounded substances [Drago 2009b]⁶. He linked these weight differences to weight-mass conservation. This was a trivial law according to Cusanus, since God pervades matter, which therefore cannot change in quantity [Crombie 1959, 296], [Drago 2009b].— One may object that Cusanus' book constitutes a mere program of research, since he never practised experimental science to the point of obtaining scientific results (he however invented the hygrometer). Yet, recall that at least among the musicians a very ancient and widespread practice obtained new results from artificial

6. Kuhn devotes a page to stressing the importance of physical magnitude weight for the birth of chemistry [Kuhn 1957, 262]. Yet, strangely enough he attributes this influence to Newton.

instruments. It is not by chance that Galileo's father, his brother and he also were musicians. Moreover, through the method which Cusanus had described, several physical laws were established, first those of acoustical instruments. In particular, Galileo's father, Vincenzo, inductively inferred from experiments on the string tensions obtained by charging weights more accurate and general laws than Pythagoras' law [Cohen 1984, 84].⁷ Hence, Cusanus did not need to put his program into practice, since artisans were already implicitly applying it to several kinds of phenomena.

4 The dialectic experiment/ mathematical hypothesis. Its historical birth and Galileo's determinant achievements

A century and a half after Cusanus, Galileo extensively applied the experiment/hypothesis dialectic to several fields of physical phenomena. However, Galileo had a particular conception of this dialectic, as his celebrated words emphasize:

Philosophy [read: science] is written in that great book which ever lies before our eyes—I mean the universe—but we cannot understand it if we do not first learn the language and grasp the symbols, in which it is written. This book is written in the mathematical language, and the symbols are triangles, circles and other geometrical figures, without whose help it is impossible to comprehend a single word of it; without which one wanders in vain through a dark labyrinth. [Galilei 1890, vi, 232]

This mathematical language is the same as God's; and whereas we have to go step by step, He sees all in the blink of an eye. Hence, geometry was already written in the world since eternity. In this conception of Galileo we recognize—as Koyré correctly emphasized, scandalizing the positivist historians of science—an ontology: not only is mathematics outside our minds (which however can understand it), it is essentially inherent in both reality and God; that is, Galileo considers geometry such a perfect theory as to be a supernatural theory that structures reality.⁸ This Platonist conception of mathematics

7. He improved Pythagoras's law, successfully suggesting the quadratic and even more complex relationships between two variables; this was a mathematical breakthrough, which later his son Galileo performed for the law regarding the case of the falling bodies; an initial law stating a linear relationship between space and time, was discarded by Galileo in favour of a law stating a square relationship.

8. For the same reason, Koyré assumed a prejudice of a mathematical-physicist, i.e., an absolute accuracy in the results of measurements [Koyré 1961]. Therefore, he opposed the historians (e.g., Crombie) whose conceptions of the history of science do not attribute a basic role to actual infinity.

influenced the entire history of theoretical physics. The subsequent physical theory, geometrical optics, made use of points at infinity as if they were ordinary points. Moreover, Newton's mathematical technique was infinitesimal analysis, which is informed by a Platonist philosophy of infinitesimals. In the 19th century, this conception reached a climax through the dominant role played by mathematical physics, a role which in 20th century was countered by the discovery of quanta and then symmetries.

Mathematics indeed may be joined with experiment also according to an instrumental conception of it, as is the case in both chemistry and thermodynamics [Drago 2009b, 2012a]. In fact, a century and a half before Galileo, Cusanus had suggested an alternative philosophical conception of mathematics, which was summarized by McTighe [McTighe 1970]. According to him, ideal mathematical entities are built from reality (and not, as Galileo claimed, recognized as the ideal elements of reality), i.e., they are constructs of our minds. Whereas ordinary knowledge of reality is always approximate and hence it is not subject to the principle of non-contradiction (through a sharp and exhaustive division into true and false), instead, mathematics is perfect knowledge because it enjoys a specific feature; it strictly obeys this principle. Owing to this logical property, mathematics is considered by Cusanus the most certain knowledge we have; for this reason, we apply it to understanding reality. Moreover Cusanus conceived numbers as the explicit forms of unity, i.e., a number enjoys the property of being both as great and as small as it can be; thus it is through numbers that all things are best understood, i.e., referred to unity. For these reasons, the creation of our minds, mathematics, can direct our interpretation of reality. Indeed, by combining experimental data with mathematics the *intellectus* conjectures the mathematical formula of a physical law. This Cusanus' way of describing the relationship between mathematics and experiment more adequately than Galileo's previous quotation describes the activity of an experimental physicist (e.g., in such a way—by means of a conjecture—the law of falling bodies actually originated in Galileo [Wisan 1974, 207–222]). Furthermore, Galileo knew only the mathematics of geometry and proportions and did not share in the effort—elicited, e.g., by his disciples Cavalieri and Torricelli—to extend the realm of mathematics [Drago 2003]. Also for this reason, the theory of mechanics was born later, through Newton, since it required the invention of a very different mathematics, calculus (just as at present time the advances in theoretical physics require the new mathematical technique of symmetries). Before Galileo, Cusanus had, on the other hand, been able to expand the mathematics of his time (geometry and proportions) to new achievements (we will see them in the next section).

I conclude that Cusanus offered to theoretical physics a more adequate metaphysics of the scientific methodology than Galileo's, owing to latter's Platonist view of mathematics. Yet, most historians disregarded the entire metaphysical and mathematical change brought about by the introduction of the scientific method. For an instance, Cohen gave a totally objectivist

definition [Cohen 1984, 85–86]. The experimental method developed by Galileo enjoys the following two features:

- it is *mathematical* in that the relationships between the parameters are quantitative, in that the proofs are geometrical, and above all the properties of falling and projecting bodies are logically derived from a set of a priori postulates;
- it is *experimental* in that not [only] daily experience but nature subjected to artificial manipulation provides both the starting point and the final empirical check of the axiomatic system.

Cohen's words "a priori postulates" and "axiomatic" attribute to Galileo's thinking the choice AO; in the following section 6, I will disprove it. I remark that all Cohen's features of the scientific method are present in Cusanus, including "the final empirical check", in the cases of a physician wanting to cure a patient and a musician wanting to perfect his musical instruments [Cusanus 1450, 622–623]. Yet, his illustration does not follow a precise order, somewhat inattentive and also fanciful. It is apparent that this deficiency of Cusanus is due to his lack of experimental practice, which he left to others (e.g., artisans, musicians).⁹ The reason is that Cusanus is interested in exploring only the mind's faculties, not natural phenomena. Hence, he sees the elements of knowledge but he does not apply them to producing knowledge from finite reality; he is too interested in infinite reality. At most, he is interested in physical principles; e.g., the principle of relativity (through the celebrated observation that over a ship sailing in a calm sea the phenomena are the same as those on the land), the impossibility of a perpetual motion, the inertia principle [Cusanus 1462–1463].

Galileo played a key historical role in the introduction of the dialectic experiment/mathematical hypothesis, since i) he qualified this dialectic, ii) he introduced the mathematical description of the evolution of phenomena (kinematics in space and time); iii) through it, he established breakthrough theoretical laws, in particular the decisive experimental law of falling bodies, which described reality in contrast with everyday experience. This result established a new kind of truth, since it was objective in two respects, i.e., the experimental and the mathematical joined in an objective unity. Moreover, iv) he decisively propagandized his historically important advances notwithstanding the harsh opposition of Aristotelian philosophers of his time; v) he practised and made this method productive to the extent of achieving an essential part of a mechanical theory; vi) his great and long activity produced so many scientific results that it eventually established a stable tradition of experimental research that, subsequently followed by several other scholars,

9. He contented himself with illustrating his experimental method by describing the simplest artificial instrument, a wood spoon [Cusanus 1450, IV, chap. II, IV].

systematically accumulated new commonly recognized results.¹⁰ From all the above I conclude that Cusanus anticipated all the elements of Galileo's experimental method; however, this consideration in no way detracts from Galileo's glory in having organized all these elements into a consistent set of rules and systematically applied them to a multitude of natural phenomena, so that he established a new tradition in science. Rather, the above anticipations by Cusanus present the historical birth of this dialectic as a less clear-cut change than that commonly presented by historians in the following words: "Galileo had no forebears and stands apart from history" [Wallace 1998, 27].

5 The second dialectic: potential infinity/actual infinity. Its historical birth

Ancient Greek mathematicians deliberately avoided the use of the infinity. Late, first Cusanus introduced a conception of the infinity into mathematics¹¹ and developed it (e.g., he presented the intuitive notion of a mathematical limit as a series of multilateral polygons approaching a circle). Owing to this innovation, Cassirer considered Cusanus to be the first modern philosopher of knowledge [Cassirer 1950].¹² In addition, I have discovered [Drago 2009a, 2012a] that he defined (without mathematical formula, yet through exact words, the infinitesimal: "of which there cannot be a lesser [positive] number" [Cusanus 1440, I, 4, 11], i.e., the basic notion, the hyper-real number, of non-standard analysis [Robinson 1960, chap. X]. On the other hand, two centuries after Cusanus, Galileo's conception of mathematics was mostly that of the ancients, also regarding infinity. In the book concluding his scientific career [Galilei 1638, First Day] he discusses the two ideas of infinity (AI and PI) in order to understand how to apply one of them to physics. Having dissected the problem, Galileo concludes that in mathematics one cannot define the usual arithmetical operations on infinite objects. Moreover, by recognizing that he is unable to decide which kind of infinity he has to choose and even when one

10. This appraisal on Galileo's contributions to this dialectic is comparable with similar appraisals suggested by eminent scholars on Galileo's method, i.e., Wisan, Machamer and McMullin [Wallace 1992, 7–12].

11. It was recognised also by G. Cantor, the inventor of the Theory of (infinite) Sets [Cantor 1883, fn. 2].

12. "This one's stand facing the problem of knowledge of Cusanus does him the first modern thinker" [Cassirer 1978, I, 39]. He presents Cusanus as: "[...] the founder and champion of modern philosophy" [Cassirer 1978, I, 39]. Note also [Vanstenbeerghe 1920, 279]: "The key to the philosophical system of Nicholas of Cusa [...]—and this is very modern—is its theory of knowledge". Unfortunately, Koyré overlooked Cusanus' role [Federici Vescovini 1994], although Cusanus more than any other scholar broke—precisely the fortunate title of the major Koyré's book—the closed Cosmos of the Ancient Greeks and opened minds to the infinite Universe.

of them is useful in formulating his laws, he asks to be allowed in any case to make use of the notion of infinity in theoretical physics.

6 The third dialectic: axiomatic organization/ problem-based organization. Its historical birth

During the birth of modern science, Cusanus organized his theological theories in a different form from the Aristotelian organization, i.e., on problems (PO) [Drago 2009c, 2012b]. This point was very clear to him as a theologian: in theology to choose AO means to derive all the truths from a priori dogmas; however Cusanus wanted to solve problems, e.g., the best name of God, the double nature of Christ, the Trinity, the constitution of both the Universe and matter, the new logic, peace in the world, etc. Also his scientific programme of weight measuring is aimed at solving problems, ultimately the problem of how to acquire the knowledge of the natural world. In sum, also in scientific subjects Cusanus' choice is for PO. A century and a half after Cusanus, Galileo's theoretical formulations of his scientific results did not conform to the traditional way of organizing a theory, AO, either, which he knew well;¹³ in particular, he never appeals to some general principle from which to deduce physical laws [Clavelin 1996, 66]. Yet, he does not assume a definite position on this dialectic. In each of his last two books he illustrates their contents by alternating two kinds of organization—deductive and dialogical-inductive, the latter recalling the dialogues of Aristotle's adversary, Plato. In sum, he did not choose a specific model (also because he did not complete any theory). As a matter of fact, Galileo's *experimental* method of producing science is in contrast with the Aristotelian model of organizing a scientific theory (see its elementary presentation in [Beth 1959, § 1.2]), but subsequent scientists, owing to his inconclusive position on this subject, eventually lessened the import of his innovations; they changed only one postulate of the Aristotelian model, the evidence postulate, which was attributed no longer to axioms, but to the data of the theory.¹⁴ In sum, the result was a mere reform of this organization, not an alternative model to AO.

13. It is a merit of W.A. Wallace to have emphasized Galileo's *Juvenilia* manuscripts concerning the illustration of Aristotle's apodictic organization of a theory. Yet, Wallace wanted to exploit this fact in order to link Galileo to Aristotelian philosophy, although Galileo's last two books manifestly show that he did not share Aristotle's model of organization [Wallace 1992], [Coppola & Drago 1984].

14. As a consequence the dialogical parts of Galileo's last two books have been misinterpreted as no more than odd presentations of the subject to the reader.

7 The third dialectic: classical logic/ non-classical logic. Its historical birth

The dialectic regarding the kind of organization is formalized by means of two different kinds of logic, respectively classical logic—governing the deductions of a AO theory—and non-classical logic—governing the inductions of a PO theory. Some scholars have remarked that the logical ways of arguing of both Cusanus and Galileo are different from those of classical logic. These differences were attributed to their attendance at the Padua school of logic, where Zabarella made the greatest effort to theorize a different way of reasoning from that of Aristotle. Actually, after the Padua period, Cusanus wanted to found a new kind of logic and he was successful in this aim. He believed that Aristotle’s logic represented a specific activity of the *ratio*, a particular way of arguing of the *mens*; yet, according to him there exists another activity of the mind, i.e., the *intellectus*, which generates *coniecturae* according to a new kind of logic. He wanted to characterize the specific laws of this new logic. He suggested a celebrated “coincidence of opposites”, which he later abandoned as ineffective. Rather he implicitly changed logic. An accurate inspection of his texts shows that he made use of the characteristic propositions of non-classical logic, those for which the double negation law fails; i.e., the doubly negated propositions whose corresponding affirmative propositions lack evidence (DNPs). Through them he solved his main problem—the best to name God—, by suggesting names pertaining to non-classical logic (Not-Other, *Posse=est*). Moreover, he was capable of developing the logical arguments, which are specific to the ideal model of a PO theory, i.e., *ad absurdum* arguments [Drago 2012a].

Some authors have remarked, on the other hand, that Galileo paid little attention to logic. In particular, he never attacked Aristotle’s legacy in logic. Moreover, his last two books present a strange logical approach; when expounding theorems, he makes use of propositions in Latin in accordance with classical logic;¹⁵ when illustrating his investigations he makes use of a Platonist-like dialogue among three people all speaking the vulgar Italian language; an inspection of the logic of these dialogues recognizes several DNPs and hence an implicit use of non-classical logic [De Luise & Drago 1995, 2009]. As an instance, let us consider the first 50 pages of his *De motu locali*. An examination of them shows that:

1. The parts of his text written in Latin and concerning formal theorems do not include DNPs;
2. The other parts—written in vulgar Italian—include more than 100 DNPs;

15. On this basis, Wallace states that “if one takes reliance on Aristotle’s logical canons to be the sign of a Peripatetic, one [Galileo] can rightfully be called a Peripatetic himself”, [Wallace 1998, 51]. Yet he ignores the inductive parts of Galileo’s theoretical work.

3. However, some of them are uncertain because some are dubious;
4. Moreover, there is no DNP in the part illustrating rectilinear uniform motion;
5. Instead there are around 90 DNPs in the 25 pages dedicated to solving the problem of naturally accelerated motion;
6. The latter DNPs compose 8 cycles of reasoning, which are illustrated by table 1;
7. Three of these cycles are *ad absurdum* proofs;
8. His reasoning eventually obtains a DNP which is a universal predicate [Galilei 1638, 190–191]; it represents an hypothesis solving the problem;
9. Which then the author translates—by merely dropping the two negations of this DPN—into the corresponding affirmative proposition in order to deductively derive from it, now considered as a postulate subject to classical logic, all possible consequences to be tested by experiment. Galileo is the only scientist apart from Einstein to have illustrated this logical step through admirably precise words [Einstein 1905a, 891]:

Let us then, for the present, take this as a *postulate*, the absolute truth of which will be established when we find that the inferences from this *hypothesis* correspond to agree perfectly with *experiment* [Galilei 1638], [Galilei 1638, 183, period before Theorem 1, Proposition I of the Naturally accelerated motion; emphasis added]¹⁶

However, Galileo ignored the alternative logical nature of both the DNPs and the *ad absurdum* arguments, maybe because his way of reasoning by means of DNPs was sometimes invalid (actually, it was invalid, according to recent studies, also in the deductive reasoning) [De Luise & Drago 2009, fn.15]. I conclude that, his genius was capable of achieving exceptional, but irregular results in logic.

16. This author's translation is justified by appealing to the principle of sufficient reason, which Galileo applies, but not in the right place, in the conclusion of the next cycle of reasoning: "where the mobile moves indifferently to either the motion or the rest, and by itself has no inclination to the motion to no location, nor any resistance to be removed; because in the same way it is impossible that a heavy body or a compound of such bodies naturally increases its height, by going away from the common center towards which all heavy bodies converge, in the same way it is impossible that it spontaneously moves, if such a motion does not approach the above-mentioned common center" [Galilei 1638, 203].

1°	2°	3°	4° ($3 \rightarrow \perp$)	5° (the case $v = 0$)	6° ($v = ks \rightarrow \perp$)	7°?	8°
Hp of a law $v = kt$	Hp of infinite degrees of velocity	Objection: Is time infinite?	Refutation: As many degrees of velocity as the degrees of time	Refutation of the objection: Even in the case $v = 0$?	Falsity of the other law $v = ks$	Laws on falling bodies on inclines	

Legenda:
Hp = hypothesis; $\perp$ = absurd; k = constant; s = distance; t = time; v = velocity

TABLE 1.The eight cycles of reasoning in *De motu locali*

8 Conclusion: What is Galileo’s role in this centuries long historical process?

First, one has to take into account that Galileo’s methodological change and his several scientific results were not enough to give rise to modern science because he did not achieve any complete physical theory. Rather both Cusanus and Galileo anticipated scientific theories. On this point a comparison is not easy because their advances anticipated two very different scientific theories, respectively chemistry [Drago 2009b], which was born three centuries after Cusanus; and mechanics, which was born a few decades after Galileo’s death. Strangely enough, neither anticipated geometrical optics; it was instead anticipated by Grosseteste four centuries before its birth, which occurred in the last period of Galileo’s life. Hence, an appraisal on Galileo’s role cannot refer to a complete scientific object, but only to the elements of physical theories. We will consider their most important elements, i.e., the foundational choices.

Let us compare Cusanus and Galileo with regard to the two theoretical dialectics. Cusanus was the first to conceive mathematics non-Platonically and he improved the mathematics of the time by first introducing infinity; and regarding infinity, he formulated, albeit in verbal terms, the notion of limit in PI mathematics and a basic notion of AI mathematics. First Cusanus made them manifest through the invention of new mathematical notions and new (theological) theories which relied on the alternative choices—respectively, AI and PO—to the dominant ones, PI and AO. He introduced a new logic, which he qualified as non-Aristotelian, at present recognized as intuitionist. Moreover, he intensively discussed the two dialectics regarding infinity and logic. Already Cassirer stressed that no one more than Cusanus, after the first of his main books, *De docta ignorantia* of 1444, accomplished such a metaphysical change [Cassirer 1950, 277].

One century and a half after Cusanus came Galileo, who discussed the relevance of the two kinds of infinity in theoretical physics, any way he did

not decide whether and how this dialectic had to be solved by a scientist; but he did not want to appeal to AI. In addition, he knew well both the organization of a theory and the logic suggested by Aristotle. He looked for an alternative logic, yet his Paduan period did not lead him to suggest a specific novelty; although as a matter of fact he adapted some of his theoretical works to the characteristic features of the model of a PO theory, e.g., the DNPs, he considered an alternative organization of a theory to be no more than a Platonic dialogue.

I conclude that regarding the two theoretical dialectics Cusanus was more advanced than Galileo.

In conclusion, it is *to Galileo glory to have given birth to science in its first dialectic through systematic experimental practice*; he was aware of the other two theoretical dialectics, but was inconclusive about them. For these reasons he has rightly been characterized as “The first modern scientist and the last of the ancient Greeks”. Cusanus played a somewhat complementary role; regarding the first dialectic, he has suggested only a programme for an experimental science of nature; however, regarding the latter two theoretical dialectics he preceded and was more advanced than Galileo, so that he anticipated the discovery of them. Which allows us to characterize Cusanus by means of a similar definition to the one above: “The last of the medieval scientists and the first philosopher of the foundations of modern science”.¹⁷ Although Cusanus had a considerable influence on the Italian intellectual milieu, e.g., on Leon Battista Alberti, Leonardo da Vinci and Giordano Bruno, no direct connection with Galileo is known. Had Galileo assumed Cusanus’ priorities, he would represent the culmination of a long intellectual effort started more than one century before. If, alternatively, he did not know Cusanus’ works, Galileo’s genius proves to be even greater, but also little explained.

After Galileo, without any discussion about the said dialectics, Newton decided the theoretical dichotomies with a pair of choices, which became a paradigm throughout the two subsequent centuries of development of theoretical physics. Due to this fact, a winter set in thinking about the two theoretical dialectics; in this winter only two scientists, i.e., L. Carnot [Carnot 1803, xiii–xvii, 3] and independently Einstein [Einstein 1905], made manifest the four choices, without receiving attention on this subject [Drago 2013]. Hence, one more merit has to be attributed to Galileo, i.e., at the very beginning of modern science to have presented through his discussions almost all the foundational issues of the scientific enterprise which have been re-discovered only after a very long period of time.

17. Koyré acknowledge him as “the last great philosopher of the dying Middle Age” [Koyré 1957, 6]. Cassirer as “the first modern philosopher” [Cassirer 1950, 31–56].

Acknowledgments

I am grateful to Prof. David Braithwaite for having revised my poor English and to an anonymous referee for some important suggestions.

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