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“It Is Impossible to Deceive Nature”. Galileo’s *Le mecaniche*, a Bridge between the Science of Weights and the Modern Statics

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“It Is Impossible to Deceive Nature”. Galileo’s *Le mecaniche*, a Bridge between the Science of Weights and the Modern Statics

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Résumé: *Il est impossible de tromper la nature.* C’est l’avertissement que Galilée adresse aux « ingénieurs ignorants » qui étaient convaincus que les machines pouvaient vaincre la nature en réalisant l’impossible. Dans *Le mecaniche* il montre que les mouvements mécaniques ne peuvent pas se produire contre la nature. Les machines doivent obéir à certaines lois de la nature auxquelles on ne peut pas déroger, comme le *principe de compensation* qui établit une liaison entre la force motrice et la résistance déplacée; formulé en termes de mouvements virtuels, il devient le principe fondamental d’une nouvelle approche de la dynamique, au moyen de laquelle Galilée construit une nouvelle science de l’équilibre, première forme de la statique moderne.

Abstract: *It is impossible to deceive nature.* This is Galileo’s warning against the “ignorant engineers” who were convinced that machines could overcome nature by undertaking impossible projects. In *Le mecaniche* the Pisan scientist shows that mechanical movements cannot happen against nature. In fact, machines must obey some inviolable laws of nature like the *compensation principle*, which establishes a connection between the moving force and the moved resistance. This principle, formulated in terms of *virtual movements*, becomes the fundamental principle of a new approach, the dynamic one, by means of which Galileo constructs a new science of equilibrium, which shapes the first form of modern *statics*.

1 Introduction: the two versions of *Le mecaniche*

On September 26, 1592, the “Serenissima Repubblica” of Venice assigned Galileo the chair of Mathematics at the University of Padua.¹ On accepting the position, Galileo left the University of Pisa, where he had taught for three years, and moved to Padua, where he remained for 18 years. Galileo himself defined these years as “the most beautiful” ones of his life. During this time, his prestige, as both a scientist and a teacher, grew beyond all bounds. Besides public teaching, he also provided private teaching in his house to numerous scholars coming from every part of Europe. At the University, he held two courses every year, as we can see in the *Rotuli artistarum dello Studio di Padova Pars Prior 1520-1739* [Viviani 1717], a very precious source in the University of Padova Archive, despite suffering from lacunae and incompleteness. By means of this and other sources, we know that during his debut year (1592-1593), Galileo taught “science of the fortifications” and “mechanics”. Vincenzo Viviani (1622-1703), who assisted him in the final years of his life, writes in his *Racconto istorico della vita del signor Galileo Galilei*² that during his stay in Padova, Galileo

[...] wrote for his students some treatises, among which one on Fortifications, according the use of that times, one on Gnomonic and practice Perspective, an epitome of the Sphere and a treatise on Mechanics which circulates in manuscript form and that P. Marin Mersenne translated into French and, in 1634 published in Paris, and lastly, in 1649, Cavalier Luca Danesi published in Ravenna. [Viviani 2001, 39]

Viviani’s indications are so clear that they leave no room for interpretation: the text on mechanics he is speaking about is the work *Le mecaniche* that Antonio Favaro (1847-1922) published in 1891 in the second volume of the National Edition of Galileo’s *Opera* [Galilei 1890-1909]. In the National Library in Florence, there are two autographed manuscripts of Viviani’s *Racconto* and in one of them, close to this quotation, the author, in his own hand, added the following gloss:

In 1593 he wrote *Mechanics* and other things. [Galilei 1890-1909, XI, cc. 22–68]; [Viviani 2001, c. 35v]

This evidence would lead us to conclude that in 1593 Galileo wrote “the treatise on mechanics, which circulates in manuscript form”, *Le mecaniche*. Yet as I demonstrated in my essay, *Tra la scienza dei pesi e la statica. Le*

1. A chair left vacant since March 1588 when his regular professor, Giuseppe Moletti (1531-1588), passed away.

2. This work was published for the first time in [Viviani 1717], but here we will quote from the recent edition [Viviani 2001].

mecaniche di Galileo Galilei, published together with the critical edition of this work in 2002 [Galilei 2002, IX–CXLIV], the date of 1593 refers not to *Le mecaniche*, but to another text, which, in an abridged way, deals with the same matter. Evidently, Viviani knew only the text published for the first time in French by Marin Mersenne (1588–1648), so he claims that it was the one Galileo wrote in 1593.

Cornelio Will, who was the archivist and advisor of the Prince of Thurn-Taxis, brought the existence of this shorter version to Favaro’s attention. In April 1898, Will wrote to the Italian professor, indicating that in the Prince’s library, he had discovered a code containing some Italian manuscripts, one of which was attributed to Galileo. Favaro verified that, alongside the text of *Le mecaniche*, there was a second text, entitled *Delle meccaniche lette in Padova dal Signor Galileo Galilei l’anno 1594* [Favaro 1889]. This text concerned nearly the same matters, but was written in a more concise form. In my aforementioned critical edition, I considered them to be two versions of the same work, and without much imagination, referring to their length, I called them respectively *versione breve*, that is, *short version*, and *versione lunga*, *long version*. I also demonstrated how the text of the *short version* relates to the course of mechanics Galileo held during the academic year 1592–1593, and how the *long version* concerns the lessons from the academic year 1598–1599.

From a formal point of view, the two versions are characterized by the different development of their texts: the *short version* is concise to the point of almost being reduced to the essential concepts with only some demonstration. The *long version*, however, ranges widely with great linguistic and stylistic accuracy, and includes exhaustive and beautiful demonstrations. This helps us understand that, while the first was merely a lesson guide Galileo used in the first year of his teaching, the second was the result of a well-thought-out and more accurate revision whose aim was the completion of a real treatise on mechanics. We can recognize this fact by comparing the introductions of these two works: in the *short version* it runs to a few lines concerning the definition of mechanics and the list of five simple machines; in the *long version* there is a very interesting dissertation on mechanics including unexplored aspects on the theory of machines. One of the most relevant aspects of the *long version* is the fact that it had been conceived as a geometrical treatise. The introduction is followed by a chapter of definitions and axioms, which is lacking in the *short version*. The presence of this chapter is clear proof of Galileo’s intent to compile a rigorous treatise in which any concept had to be introduced if it had not been defined and explained beforehand, similar to Euclid’s *Elements*. This desire to be clear and precise stands out also in the insertion (after the treatment of the lever) of the chapter “Alcuni avvertimenti circa le cose dette” [Some notices about the aforesaid things], also missing from the *short version*. In this chapter, Galileo sheds light on a few important questions concerning the angular lever and the second-class lever, which are not in any other treatise of that time. However, in comparing the two texts, an analysis of the technical-scientific contents shows that we are examining two successive phases in the

development of the same ideas. The contents of the *long version* are the same as those of the *short version*—apart from the sections concerning the so-called *Baroulkos*, namely, a train of shafts into a wheel, and concerning the perpetual screw, which are present only in the *short version*. In the *long version*, Galileo deals with these subjects in a wider and more detailed fashion, adding a few pertinent and remarkable demonstrations, including many good examples, and he introduces some innovative ideas on the theory of machines. But the most peculiar aspect that distinguishes this version and makes it a new original treatise in mechanics is its systematic use of the concept of the *moment*.

2 The definition of mechanics in the *short version*

One of the most important questions we want to emphasize is the fact that Galileo provides a new definition of mechanics within the few lines of the *short version*:

The science of mechanics is the faculty that teaches us the reasons and makes us understand the causes of the miraculous effects that we see happen with some tools, in moving and lifting heavy weights with a small force. [Galilei 2002, 5, ll, 1–4]

This definition marks a caesura with the Aristotelian tradition still alive at that time. At the beginning of his Μηχανικά Προβλήματα, the pseudo-Aristotle writes:

Among the events that occur in accordance to nature, those of which the cause is unknown arouse wonder, while, among the events that occur against nature, arouse astonishment all those are realized with art for the benefit of mankind. [Aristotele 2000, 847a, 11–13]

According to the theory of natural places, the pseudo-Aristotle distinguishes mechanical phenomena *according to nature* (κατὰ φύσιν), from those *against nature* (παρὰ φύσιν). He maintains that, while our advantage is susceptible to change in many ways, nature always acts consistently and plainly in the same direction. So, in many cases, nature can produce disadvantageous effects for our utility. Therefore, if we wish to do something *against nature* we must resort to τέχνη, to *art*, to the help of some tool. For this reason, the pseudo-Aristotle defines mechanics as follows:

The part of the art which comes to our aid in such difficulties. [Aristotele 2000, 847a, 17–19]

For the pseudo-Aristotle, and the long tradition that followed him and still dominated at the time of Galileo, mechanics does not explain the nature of

the bodies or their natural motions; it is *art* that, by means of machines (*artificials*), allows us to make things *against nature* (παρὰ φύσιν) and that for this reason arouses wonder. The numerous Renaissance commentators on the Μηχανικά Προβλήματα have interpreted the above-mentioned opening passage of this work in various ways, according to the different ways to translate παρὰ φύσιν. These interpretations are very important in establishing their epistemological position towards the Aristotelian theory of natural places.

In my previously mentioned critical edition, I pursued a wider and deeper analysis of these positions, concluding that only Galileo fully develops the theory of natural places. Galileo first subjected this theory to severe criticism in *De motu* [Galilei 1890-1909, I, 243–419], but only in the two versions of *Le mecaniche* does he complete his project to revise and re-establish the bases of the mechanics.

3 The *compensation principle* in the *long version*

As we have seen, in the *short version*, Galileo defines mechanics as the science that teaches the reasons and explains the causes of the effects that may appear to be miracles only to those who ignore the true laws of nature. It explains arcane ideas concerning “miraculous” phenomena connected to the use of tools, and causes them to lose any attribute of irrationality. Galileo thus divests mechanics of any metaphysical element, and shows that the machines do not possess any supernatural virtue but operate according to precise laws of nature.

In the *long version*, he makes it even more explicit that nothing can happen which nature does not allow. Because of the ignorance of the laws of nature, he writes:

I have seen the generality of mechanicians deceive themselves in going to apply machines to many operations of their own nature impossible [...]. Of which mistakes I think to have understood the cause to be essentially the belief that these artificers had and still have to be able to lift great weights with a small force, in a certain manner deceiving nature, whose instinct, yet unchanging constitution it is, that no resistance can be overcome but by a force that isn’t more powerful than it. [Galilei 2002, 45, II, 8–17]

This passage is remarkable. Here Galileo plainly reaffirms that nature cannot in any way be deceived, that it is impossible to operate against the natural order. The machines have to observe the unavoidable law that “no resistance can be overcome but by a force that isn’t more powerful than it”. So where does the benefit of the machines lie? In order to answer this question Galileo explains that in the working of the machines we must consider four things:

The first one is the weight to move from place to place; the second is the force or power that has to move it; the third is the distance between both terms of the motion; the fourth is the time in which this change must be made; and it is the same thing if instead of this time we consider the quickness and the speed of motion, as we define that motion faster than another which covers that distance in a shorter time. [Galilei 2002, 45–46, ll, 28–34]

Previously in the *short version* Galileo had pointed out that, in the working of a machine, not only were the moving forces in play, but the speed of the moved resistance also had to be considered:

But it is necessary to notice that the more the effort is in using the lever, the more, on the opposite, we spend time; and the less the force is with respect to the weight, the greater is the space through the force will move with respect to the space through which the weight moves. [Galilei 2002, 7, ll, 75–79]

The working of a lever, and of all the other simple machines that rely on it, follows a principle according to which force, resistance, time, space, and speed continuously compensate each other: if the power increases, the speed decreases because, in moving the resistance through a given space, the force crosses a space longer than it in the same time. This is the reason why we call this statement the *compensation principle*. In the introduction to the *long version*, Galileo explains that, with no machine, it is still possible to move a great weight with a small force from one place to another by dividing it into a certain number of parts, with no part being superior to the moving force, and transferring them one by one until the whole weight is carried to the assigned place. He firmly remarks, at the end of this work, that nobody can affirm that a great weight has been transported by a force less than it, but only that it has been moved by a force that is reiterated many times along that space which the whole weight, all together, would have traversed only once. In so doing, it is clear that the velocity of the moving force is many times superior to that of the moved weight, as this weight is superior to the force indicated. While this force has crossed the given space many times, the whole resistance has crossed it only once. Thus, Galileo correctly concludes:

Therefore, we ought not to affirm that a great resistance has been overcome by a small force, contrary to the constitution of nature. Then only we will be able to say that the natural constitution has been overcome, when the lesser force would transfer the greater resistance with the same speed of motion with which the force moves; that we absolutely affirm to be impossible to be done with any imaginable machine. [Galilei 2002, 7, ll, 75–79]

The *compensation principle* appears again and again in *Le mecaniche*, from beginning to end. Upon reading the introduction, we soon recognize that one

FIGURE 1.

of Galileo's previous aims was to show that every machine is subject to this same rule. In fact, this principle appears to be a thread running through the entire treatise, which Galileo expressly reasserts when he concludes the discussion of each machine considered. At the end of the exposition on the balance and the lever in the *long version*, he remarks that

The benefit drawn from this instrument is not that of which the common *mechanics* are persuaded, that nature could be overcome and, in a certain manner deceived, a small force overpowering a very great resistance by means of a lever, because we will demonstrate that, without the help of the length of the lever, the same lever, the same force, in the same time obtains the same effect. [Galilei 2002, 55, ll, 392–398]

Here, he shows the real function of the tools, thereby demonstrating that nature cannot be deceived. Given the lever BCD (Fig. 1) with the fulcrum at C , the force applied at D and the resistance at B , let us suppose the distance CD , for example, to be five times the distance CB . It is clear that the force is able to lift a weight five times greater than itself. Now let this lever move until it assumes the position ICG . The force has now passed through the space DI while the weight has crossed the space BG . Since $DI = 5CB$ and $B\hat{C}G = D\hat{C}I$, then necessarily $ID = 5BG$. If we then consider the distance CD at point L , so that $CL = BC$, the same force applied at L will be able to lift from point B only at an equivalent of one fifth of the weight that was placed there at first. Now given that $CL = BC$, the distances LM and BG , crossed respectively by the moving force and the resistance, will be equal. Reiterating this action five times, we may observe the same effect: the whole weight will be lifted from B to G . At this point Galileo concludes:

But the repeating of the space LM is certainly more than the single measurement of the space DI , that is five times LM . Therefore, the transferring of the weight from B to G requires neither less force, nor less time, nor a shorter way if the weight were placed at D , than it would need if the same force had been applied at L . [Galilei 2002, 55, ll, 392–398]

Therefore, the benefit derived from the use of this tool is only the possibility to move a given body all at once. But the time the tool itself takes in moving the weight will be just the same it would take if, having divided the space into a certain number of parts, each one equal to the given force, this force had transported them the same distance one at a time. After having explained the law of the shaft into the wheel, Galileo considers the case of a winch in which the diameter of the wheel is ten times that of its shaft; so this tool is able to lift a weight ten times that of the force applied.

FIGURE 2.

He explains that, when the force moves once along the circumference FCG (Fig. 2), the shaft EAD , to which the weight is tied, winds around the rope and lifts the weight in completing only one turn. Therefore, the weight H will cover only the tenth part of the way covered in the same time by the moving force. At this point, reproducing the same reasoning of the introduction, Galileo points out:

If therefore, by means of this machine, the force for moving a resistance greater than itself, along a given space, must of necessity move ten times as far, there is no doubt that, dividing the weight into ten equal parts, each of them will be equal to the force, and consequently, it might have been transported one at a time, as great a space as that which itself will move. So that, making the journeys, each equal to the circumference EAD , it will not have gone any further than if it had moved only once around the circumference FGC and had carried the same weight H to the same distance. [Galilei 2002, 50, ll, 545–550]

In concluding, he reasserts that the benefit derived by the use of this tool is only that it is possible to carry the given weight together across the given distance, but not with less labor, or with greater speed, or a greater distance that the same force might have used if carrying parcels. The validity of this

principle was also the focus for the second-class lever and the block and tackle. In the first case, referring to Fig. 3, Galileo writes:

And by moving the weight, with the lever used in this manner, it is gathered in this also, as well as in the other tools, that what is gained in force is lost in speed. In fact, when the force C raises the lever and transfers it to AI , the weight crosses the space BH , which is as much lesser than the space CI passed by the force, as the distance AB is lesser than the distance AC ; that is, as the force is less than the weight. [Galilei 2002, 61, ll, 600–606]

FIGURE 3.

In the second case, in determining the behavior of a block and tackle with four pulleys (Fig. 4), he says:

And we will likewise note, that to make the weight ascend, the four ropes BL , EH , DI , and AG ought to pass, whereupon the mover will be to begin, as much as those four ropes are long; and yet, nevertheless, the weight shall move but only as much as the length of one of them. So that we say this by way of advertisement, and for confirmation of what has been many times spoken, namely, that look with what proportion the labor of the mover is diminished, the length of the way, on the contrary is increased with the same proportion [Galilei 2002, 65, ll, 756–765].

4 The Definition of *moment*

It should be noted that in the case of the inclined plane, and consequently of the screw, Galileo also shows that the *compensation principle* is valid. But in order to properly understand the role this principle plays in Galileo’s theory of the inclined plane, it is necessary to first introduce the concept of *moment*. In the chapter entitled *Diffinizioni* [Definitions], Galileo defines *gravity*, *moment* and *center of gravity*. Of the first, he writes:

Therefore, we call gravity the propensity of moving naturally downwards, which in solid bodies is caused by the greater or less

FIGURE 4.

quantity of matter, whereof they are constituted. [Galilei 2002, 48, ll, 139–143]

As concerns the second, he writes:

Moment is the propensity of descending, caused not so much by the gravity of the moveable, as by the disposure which diverse grave bodies have in relation to one another; by means of which moment, we oft see a body less grave counterpoise another of greater gravity: as in the steelyard, a great weight is raised by a very small counterpoise, not through excess of gravity, but through the remoteness from the point whereby the beam is upheld, which conjoined to the gravity of the lesser weight adds thereunto moment, and impetus of descending, wherewith the moment of the other greater gravity may be exceeded. Moment then is that impetus of descending compounded of gravity, position, and the like, whereby that propensity may be occasioned. [Galilei 2002, 48–49, ll, 143–145]

Therefore, Galileo defines both *gravity* and *moment* similarly, that is, as a “propensity” to descend. The first is a natural intrinsic propensity of the bodies due to the “quantity of matter whereof they are constituted.” The second is an extrinsic propensity due to the gravity of the moving body and to

“the disposure which diverse grave bodies have in relation to one another”. It is by virtue of this “disposure” that a body with less gravity is able to balance one of greater gravity. The example of the steelyard, in which a small weight is able to counterbalance a greater one, is very appropriate. It clearly shows that this effect occurs “not because of excess of gravity, but of the distance from the point whereby the steelyard is supported”. Therefore, the *moment* is a quantity composed of gravity and distance joined together which particularly comes into play in the use of machines. In fact, “the disposure which diverse grave bodies have in relation to one another” is simply the distance between the acting power and the center of rotation of a balance, such as that of a shaft into a wheel or the fulcrum of a lever, etc. This is the definition of the *static moment* that we analytically represent as $\text{force} \times \text{distance}$.³ But, in concluding the definition of *moment*, Galileo affirms that this quantity can be composed, not only by gravity and position, but also by other elements which may cause the propensity to descend. This statement is particularly remarkable because it provides insight into the possibility of some different understandings of the definition of *moment*. Everything becomes clear in the chapter “Alcuni avvertimenti circa le cose dette”, in which he shows

how the velocity of the motion is able to increase moment in the moveable, according to that same proportion by which the said velocity of the motion is augmented. [Galilei 2002, 53, ll, 332–335]

He considers the lever AB divided in unequal parts by the fulcrum C and two weights at the points A and B such that the lever is in balance (Fig. 5).

FIGURE 5.

He has already explained that equilibrium occurs when the ratio between the weights equals the inverse ratio of the respective distances. At this point, he first points out that a very small *moment* of gravity added to one of the two extremities is enough to break the equilibrium, and then he considers how the lever moves to arrive at the new position DCE . We will first examine this

3. Really, Galileo never expressed *moment* with this formulation, as he did not define in mathematical terms any derived physical quantity.

last question, then return to consider the first crucial question a little later. In considering the motion that the weight B makes when descending into E , and that the weight A contemporary makes in ascending into D , Galileo remarks:

we shall without doubt find the space BE to be so much greater than the space AD , as the distance BC is greater than CA .
[Galilei 2002, 53, ll, 313–314]

In fact, given $D\hat{C}A = E\hat{C}B$, the ratio of the arcs BE and AD will be equal to that of the semi-diameters BC and CA of the circumferences to which, respectively, they belong. So that

the velocity of motion of the descending grave B comes to be so much superior to the velocity of the other ascending mobile A , as the gravity of this exceeds the gravity of that; and, as it is not possible that the weight A should be raised to D , although slowly, unless the other weight B moves to E swiftly, it will not be a surprise, or averse to the order of nature, that the velocity of the motion of the grave B should compensate for the greater resistance of the weight A , so long as it slowly moves to D , and the other one swiftly descends to E . And so on the contrary, the weight A being placed at the point D , and the other B at the point E , it will not be unreasonable that the first one slowly falling to A , should be able quickly to raise the other to B , recovering by its gravity what it had lost by the slowness of motion. And by this discourse we may come to know how the velocity of motion is able to increase moment in the mobile, according to that same proportion by which the said velocity of motion is augmented.
[Galilei 2002, 53, ll, 317–335]

Here Galileo gives a new definition of *moment* in which the quantities at play are gravity (by which he means force) and, instead of distance, the *virtual* velocity of the motion that gravity generates. It is a *dynamic* definition, which he explicitly came back to some years later, in the second edition of the *Discorso intorno alle cose che stanno in su l'acqua o che in quella si muovono*, published in 1612, in which he specifies:

In the opinion of the mechanicians, moment is the virtue, the force, the effect, through which the moving moves and the mobile withstands; this virtue depends, not only on the simple gravity, but on the velocity of the motion, on the different obliquities of the spaces on which the motion takes place, because a grave descending in a more oblique space makes impetus greater than in a less inclined. And, in short, whatever the cause of this virtue is, nevertheless it holds name of moment. [Galilei 1612, 6]

Together with this new definition of *moment*, he enunciates the following two principles:

1. Weights absolutely equal, moved with equal velocity, operate with equal forces and moments.
2. The velocity of the motion increases the moment and the force of the gravity so that weights absolutely equal, but joined with unequal speeds, have unequal force, moment and virtue, and the fastest one is more powerful according to the proportion of its velocity to that of the other one [Galilei 1612, 6–7].

Here he has replaced distance with velocity considering that, in relation to the definition of *moment*, these two quantities are equivalent. What legitimates this substitution? In order to show the perfect equivalence of this dynamic definition with the static one in *Le mecaniche*, Galileo gives the following example of a balance with equal arms:

Two weights of absolute equal gravity, placed in a balance with equal arms, are balanced, and no one of them inclines lifting the other one, because the equality of the distances of both from the center, on which the balance is sustained and around which it moves, makes sure that, moving the balance, these weights would cross at the same time equal spaces, that is they would move with the same speed. Therefore there is no reason because one of the two weights has to incline more than the other one, so that they equilibrate and their moments are of the similar and equal virtue. [Galilei 1612, 6–7]

This is clearly inspired by the principle of the circle by the pseudo-Aristotle. If the diameter of the circle rotates around its center at a certain angle, at the same interval of time, its extremes cross equal arcs, that is, they move with the same speed. This shows that the equality of the *moments* of equal weights at the extremes of the balance’s arms is assured both by the fact that these arms have the same length, and by the fact that they move with the same speed. The greatest velocity of a given weight increases its *moment* as the greatest distance does. In a balance ACB with unequal arms, $AC < CB$, if B rotates following the arc BE , A will cross the arc AD at the same time. Galileo very easily demonstrates that the ratio of the previously mentioned arcs is equal to that of the distances CB and AC . Since these arcs are crossed at the same time, it is clear that the speed of B is much greater than that of A , as the gravity of B exceeds the gravity of A . Now, given $BE/AD = BC/AC$, in considering the *moment*, it is possible to substitute the ratio of velocities to that of the distances. At the same time, BE/AD represents the ratio of the speeds, justifying Galileo’s conclusion:

By this discourse we may come to know how the speed of the motion is able to increase moment in the moveable, according to that same proportion by which the said speed of the motion is increased. [Galilei 2002, 53, ll, 332–335]

5 An “audacious assertion”.

The angular lever

At this point, we are able to come back to the crucial undecided matter concerning the breaking of a lever’s equilibrium. The question arises in the chapter “Alcuni avvertimenti circa le cose dette”, where, with regard to the balance *ABC* in equilibrium (Fig. 5), Galileo writes:

It is already manifest, that the one [weight] will counterbalance the other, and consequently, that if a very small moment of gravity were added to one of the weights, it would move downwards, raising the other. So that adding an insensible weight to the weight *B*, the balance would move and descend from the point *B* towards *E*, and the other extremity *A* would ascend into *D*. And as in order to make the weight *B* come down every small gravity added to it is enough, therefore if we don’t keep any account of this insensible moment, we will make no difference between one weight sustaining, and one weight moving another. [Galilei 2002, 53, ll, 299–310]

Galileo expresses the idea that, once the amount of force necessary for the equilibrium of a lever is established,⁴ in order to move or to lift the resistance it is enough to increase the power by any amount, no matter how small, which, in current mathematical terms, we would say, “converges to zero”. It is by the virtue of this idea that we can forego considering the addition of this “insensible moment” and affirm that there is no difference between the power that counterbalances the resistance and the one that moves it. Considering Galileo’s conclusion, which states that “if we do not keep any account of this insensible moment, we will make no difference between one weight sustaining and one weight moving another”, Marshall Clagett speaks of an “audacious assertion” [Clagett 1972, 179]. He is of the opinion that this idea was also implicit in the *Μηχανικά Προβλήματα* by the pseudo-Aristotle and in the *Mechanics* by Heron, in which “the same proportionality of the weights and of the distances (or of the times)” is applied either in the case of a balance in equilibrium, or of a machine in motion. However, Clagett maintains that the benefit of making it explicit for the first time decidedly has to be credited to Galileo. My opinion is that Heron, in his *Mechanics*, expressed the exact same concept just as clearly, namely, that in order to move a given resistance by means of machine, it is first necessary to determine the force counterbalancing it, and then to increase this force by a small amount. After having shown that, in order to keep a fixed pulley in equilibrium, a power equal to the resistance is needed, Heron adds:

4. Really it is implicit that this fact is valid for every tool.

If however a small amount is added to the weight, then the other weight is pulled upwards. If therefore the force moving the weight is greater than it, then this force will be strong enough for it and will move it except if any friction occurs in the turning of the pulley or a stiffness of the rope, so that they would cause a hindrance to the motion. [Ferriello, Gatto *et al.* 2016]

It is the same “insensible moment” of which Galileo speaks. The real difference between Heron and Galileo is the idea of *limit* the latter expresses. Another remarkable question Galileo examines in the aforementioned chapter “Alcuni avvertimenti circa le cose dette”, which concerns the concept of distance in considering the *moment* of the force acting on the angular lever. He explains that we must assume as distance the segment perpendicular to the direction of the force. He points out that in the balance ACB (Fig. 6), the weights G and

FIGURE 6.

H “make their impulse and would descend, in case they were freely moved, describing the lines AG and BH ”. Thus, if the arm CB rotates to assume the position CD , then the weight hanging on point D “will make its moment and impetus according to the line DF ”, given that F is the point at which CB meets the perpendicular line to it from D . Therefore, in considering the moment of the gravity on D , we must assume as distance the segment CF that is the orthogonal projection of CD on CB . Now given $CF < CB$, the moment of the gravity at D is smaller than at B .

6 The *compensation principle* in the inclined plane

At this point we are able to consider the question of the *compensation principle* in the inclined plane. But before getting to the heart of the matter, we can affirm that the treatment of the inclined plane is one of the most relevant subjects in *Le mecaniche*. In this treatise, for the first time in the history of mechanics, Galileo enunciated the exact formulation of the law which connects power to resistance in relation to the dimensions (height and length) of the inclined plane, namely, that

the ratio between the weight and the force is the same of that between the length of the elevated plan to the perpendicular height. [Galilei 2002, 11, ll, 191–192]

In the *long version*, this law is demonstrated in a very rigorous manner. In the *short version*, it is introduced and enunciated more simply as “from the light of nature and for experience”, which teaches that the lower the elevation of the inclined plane, the smaller the force necessary to raise a given weight. Starting from these considerations, in this version Galileo already comes to the remarkable conclusion that

if we had a plane without any inclination, the heavy bodies put on it would not move in themselves, but it is true that every very small force would be enough to move them from the place. [Galilei 2002, 10, ll, 163–167]⁵

After having introduced and demonstrated the law of the inclined plane, Galileo then focuses on the screw, that is, an inclined plane wound around its height. He questions whether or not, for this particular tool, the *compensation principle* occurs. In fact, he says:

It seems, that in this case the force is multiplied without the movers moving a longer way than the moveable. [Galilei 2002, 73, ll, 1079–1081]

Galileo is referring to the inclined plane ABC on which the movable weight E , joined by means of a rope EDF to another smaller weight F , is placed (Fig. 7). The equilibrium law of this device says that

$$E : F = BC : CA$$

FIGURE 7.

The weight F , falling along the vertical CB , causes the weight E to ascend along the inclined plane AC . Since the two weights are connected to one

5. Really, already in *De motu*, correcting Pappus, who had affirmed that in order to move a body on the horizontal plane a force equal to its weight is needed, Galileo had shown in an admirable way that “Quocumque mobile super planum horizonti aequidistans a minima vi movetur, imo et a vi minori quam quaevis alia vis [...]” [Galilei 1890-1909, I, 209].

another, the distance covered by E necessarily has to equal that covered by F . But Galileo observes that, for the equilibrium, we must consider not the lengths of the distances covered, but the lengths of their projections on the vertical axle, namely, as we would say today, the vertical component of the aforementioned movements. Now, while the weight F moves vertically in the direction of CB , the weight E moves obliquely along the plane AC . So, when E has covered the whole distance AC , it has crossed a vertical space equal to CB , and a horizontal space equal to AB , which is as if, in order to move from A to C , the weight would have crossed the horizontal space AB and then the vertical space CB . But since a force however small is enough to move a body on the horizontal plane, with regard to the oblique motion of the weight, we must consider only the vertical movement. Consequently, in order to move the weight on the inclined plane AC only the fraction BC/AC of the weight itself is needed. Therefore, the greatest distance AC requires a force smaller than that the shorter distance BC . Galileo's conclusion is:

It is therefore very interesting to consider by what lines the motions are made, especially in exanimate grave bodies, the moments of which have their total vigor and entire resistance in the line perpendicular to the horizon; and in the other transversally elevated and inclined they feel the more or less vigor, impetus, or resistance, the more or less those inclinations approach unto the perpendicular inclination. [Galilei 2002, 74, ll, 1101–1107]

The reference to Jordanus' *De ratione ponderis* [Jordanus Nemorarius 1952] is clear. Just in analyzing this aspect of Galileo's theory of the inclined plane, Maurice Clavelin states that:

While in *De ratione ponderis* the virtual moves occur only to provide the indirect demonstrations of certain peculiar problems, with Galileo they become an occasion to formulate a general principle to apply to all the simple machines. But it is also indisputable that the general principle so enunciated for the first time enveloped the possibility of a new exposition of the statics. [Clavelin 1968, 168–169]

There is no doubt that the *compensation principle*, formulated in terms of *virtual movements*, is a fundamental principle of the Galilean theory of machines that indicates the possibility for constructing a new science of the equilibrium, namely the statics. Galileo appears to be aware of the innovative significance of this principle. He turned to it in the already mentioned *Discorsi* to explain the phenomenon of the hydrostatic equilibrium in communicating vases. If, due to pressure, the water's level CD goes down to QO (Fig. 8), the water's column CL must necessarily rise up to the level AB , and the ascent LB will be greater than the descent GQ , insofar as the width of the vase GD is greater than the width of the pipe LC . So Galileo wonders:

FIGURE 8. Source: [Galilei 1980]

Being the moment of the velocity of a mobile compensate for that of the gravity of another one, which wonder will it be if the fast ascent of the little water CL withstands the slow descent of the great water GD ? [Galilei 1612, 16]

He explains that the same effect occurs for a balance in which one arm is 100 times longer than the other one. He reasserts once more that, by means of this tool, a weight P of 2 pounds is able to counterbalance the weight P' of 200 pounds only because, in the same time, P is obliged to cross a space 100 times greater than P' . It is interesting to remark that Galileo says that the example shown above

will be useful to get out of error some practical mechanicians, who embark on some impossible enterprises basing on a false foundation. [Galilei 1612, 16]

This does not differ at all from what he says in the introduction of the *long version*, where, after having affirmed to have seen “the generality of Mechanicians deceive themselves in going about to apply Machines to many operations of their own nature impossible”, and after having explained the real nature of the machine and the laws to which they are subject, he concludes:

These then are the benefits that may be derived from mechanical instruments, and not those which ignorant engineers dream of, with the deception of so many principles, and with their own shame, while they undertake impossible enterprises. [Galilei 2002, 48, ll, 123–126]

7 Conclusion

Galileo was truly the first author in the modern age who introduced the *compensation principle* [Gatto 2015] in a clear and unequivocal way, and who

used *virtual movements* to establish a new scientific base for the science of weights. Yet he was not the very first in the history of mechanics to do so. In ancient times, another scientist introduced this principle into the theory of machines. We are speaking of Heron of Alexandria who, in his *Mechanics*, at the end of the treatment of the *Baroukos*, writes:

This tool and those of great power similar to it are slow, because the smaller the moving force is relating to the weight to be moved, the longer is the time the work needs. The force to the force and the time to the time are in the same inverse ratio. [Ferriello, Gatto *et al.* 2016, 113–114]

We immediately realize that this statement is similar to Galileo’s. We likewise recognize that, as we have seen in *Le mecaniche*, Heron also reasserts this principle at the end of the discussion on the train of blocks and tackles and that of the levers, which are the other compound tools he considers.⁶ The strong similarity in considering and developing this aspect of the problem, the idea that the addition of an amount of force, *however small*, to the power which makes the balance is enough to break the equilibrium of a lever, and some other questions we cannot discuss here, would lead us to think that Galileo could have known the *Mechanics* by Heron⁷. Clagett excludes this possibility, sharing the common opinion that this work by Heron was unknown at the end of the sixteenth century. But Clagett himself, describing the *dynamic approach* in *Le mecaniche*, in which Galileo first uses the *virtual movements* to confirm the law of the lever and subsequently to derive the law of the inclined affirms that the similarity with Heron’s *Mechanics* is manifest.⁸

6. About the train of blocks and tackles, he writes: “It is clear that a delay occurs with this tool because the process takes the same ratio [of the Baroukos] [...] The ratio of the times equals the [inverse] ratio of the moving forces”. Similarly, in concluding the exposition of the train of levers, he writes: “Here too the delay occurs in the same ratio, because there is no difference between these levers and the shaft into the wheels”.

7. This treatise has reached us, for the first time, in 1893 through an Arabic manuscript of the 9th century, whose text, with its French translation, was published by Camille Carra de Vaux [Carra de Vaux 1983].

8. Also Roberto Marcolongo, although surprised because of the strong similarity of some theories by Leonardo with the corresponding by Heron, was convinced that it was impossible that Leonardo could have known Heron’s *Mechanics*. In fact, he writes: “Heron’s considerations on the centres of gravity of the plane figures, those on the theory of the simple machines, and particularly that on the blocks and tackles, are very important in the history of mechanics. But it is obvious that this book persisted to be unknown to the Western scholars, and, therefore, Leonardo couldn’t know it. But I think that we can also assert that, by some ways that we continue to ignore yet to day, some idea by Heron has reached the Western scientists and it is preserved there” [Marcolongo 1937, 138]. It is also interesting to note what Giovanni Vailati wrote in a letter to Ernest Mach (Crema, 3 August 1897): “The way in which this last author [Heron] enunciates and applies the aforesaid principle [the *principle of the virtual movements*] is such to suggest a period of the development of statics corresponding to that expressed in Galileo’s early works” [Vailati 1971, 115].

We have already explored the possibility that Galileo and other Italian authors of the Renaissance were familiar with Heron's *Mechanics* [Gatto 2015]. The influence of Heron's work on Galileo and its power to inspire him cannot be underestimated. The principle of *virtual movements* is the key difference between Heron and Galileo's approaches. Heron applied it to the lever, the block and tackle, and later to the shaft into the wheel, which are the machines he limited to the Archimedean principle of the balance. In so doing, he provided the evidence that confirmed the relationship between the efficiency of a machine and its related delay. He never applied this principle to the inclined plane, which he dealt with another way without making any reference to the above-mentioned simple machines. Galileo, beginning with the Archimedean law of equilibrium, followed by the concept of the moment, was able to conceive of a form of the *principle of virtual movements* as a general principle for the theory of machines, thereby granting mechanical science a new perspective. This accomplishment is so significant that *Le mecaniche* should be considered as the first modern treatise on statics.

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