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Perverted Space-Time Geodesy in Einstein's Views on Geometry

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Résumé : Une géodésie spatio-temporelle perversie résulte des notions de règles et d'horloges variables, qui sont prises pour avoir leur longueur et leur rythme affectés par le champ gravitationnel. D'autre part ce que nous pourrions appeler une géodésie concrète repose sur les notions de règles et d'horloges invariables de mesure d'unité. En fait, il s'agit d'une hypothèse de base de la relativité générale. Les règles et les horloges variables conduisent à une géodésie perversie dans le sens où un espace-temps courbe pourrait être vu comme provenant du départ de l'espace-temps Minkowskien comme un effet du champ gravitationnel sur le rythme d'horloges et la longueur des règles. Dans le cas d'une géodésie concrète, nous avons « directement » un espace-temps courbe dont la courbure peut être déterminée en utilisant des règles et des horloges de mesure d'unité (invariables). Dans cet article, nous défendrons la plausibilité que le point de vue d'Einstein sur la géométrie par rapport à la relativité générale est imprégné par une géodésie perversie.

Abstract: A perverted space-time geodesy results from the idea of variable rods and clocks, whose length and rates are taken to be affected by the gravitational field. By contrast, what we might call a concrete geodesy relies on the idea of invariable unit-measuring rods and clocks. Indeed, this is a basic assumption of general relativity. Variable rods and clocks lead to a perverted geodesy, in the sense that a curved space-time may be seen as a result of a departure from the Minkowskian space-time as an effect of the gravitational field on the rate of clocks and the length of rods. In the case of a concrete geodesy, we have a curved space-time “directly”, the curvature of which can be determined using (invariable) unit-measuring rods and clocks. In this paper, we will make the case for the plausibility of the claim that Einstein's views on geometry in relation to general relativity are permeated by a perverted geodesy.

1 Introduction

In his detailed analysis of Einstein’s 1916 review paper on general relativity, Darrigol noted that, when making the so-called Newtonian approximation and in his gravitational redshift derivation, Einstein seems to start with a “fictitious” Minkowskian space-time, and arrives at a curved space-time by considering the effect of the gravitational field on rods and clocks [Darrigol 2015, 172]. According to Darrigol, “in this view, space is originally Euclidean but it appears to be non-Euclidean when the measuring rods are affected by a gravitational field” [Darrigol 2015, 172], with an equivalent situation holding for clocks. Darrigol calls this view a “perverted geodesy”. Darrigol notes in a footnote, *en passant*, that Einstein’s reliance on this view is confirmed in his 1917 book, “where he compares the non-Euclidean geometry of general relativity with the apparent geometry of a table top of uneven temperature when gauged with dilatable rods” [Darrigol 2015, 172]. This paper starts from this “seed”. Our intention is to offer an historical reconstruction of Einstein’s views on geometry, and determine to what extent his views are permeated by a perverted geodesy. We will make the case for the plausibility of Darrigol’s view that Einstein relied on a perverted geodesy in relation to the space-time of general relativity.¹ While we will mention them, we will not focus in detail on the inconsistencies that would be brought about by this situation.

The article is organized as follows. Section 2 is a preliminary section, in which we review some basic elements of the theory—the meaning of Gaussian coordinates and the physical assumption of the “independence from past history” of unit-measuring rods and clocks—and consider a gravitational redshift derivation made by applying the equivalence principle in the case of clocks in a rotating disk.² We will see that this heuristic derivation is based on the “dynamical-like” view that the gravitational field (the rotation of the disk)

1. We consider in detail Einstein’s views on geometry up to 1921, when he published “Geometry and experience” [Einstein 1921a], which presented his view of what he called “practical geometry”, and when he wrote his Princeton lectures on special and general relativity (published in 1922). Together with the 1916 review paper and the 1917 “popular” book, these constitute his most detailed and influential writings on general relativity and geometry.

2. In its earlier formulation, the equivalence principle entails that an inertial reference frame in a homogeneous gravitational field is physically equivalent to a uniformly accelerated reference frame in a space free of a gravitational field [Einstein 1907, 302], [Einstein 1911], [Einstein 1913, 208]. Einstein gave an alternative formulation of the principle of equivalence, where we consider a four-dimensional space-time region in which special relativity is valid (i.e., in which we can adopt an inertial reference frame K); we then consider a second reference system K' uniformly accelerated with respect to K . According to Einstein, “nothing prevents us from considering [the] system K' as at rest, provided we assume a gravitational field (homogeneous in first approximation) relative to K'' ” [Einstein 1916b, 237–238], see also [Einstein 1916a, 150], [Einstein 1917, 318–319], [Einstein 1922, 317], [Norton 1985, 205–206].

affects the rate of clocks. This result goes hand in hand with the adoption of a perverted geodesy in this case. The bulk of this article comes in section 3, in which we consider the development of Einstein's views on geometry and see how, in several places, Einstein (implicitly) adopts a perverted geodesy: (1) In his rotating disk argument for the adoption of a non-Euclidean geometry due to the presence of a gravitational field (using the equivalence principle, the rotation of the disk is reinterpreted as a stationary gravitational field "existing" in relation to the rotating reference frame—the disk). (2) In the analogy with a heated marble slab in which "little rods" are differently deformed due to the temperature gradient in the table (which is an analogy to the curved space-time of general relativity). (3) In his reference to the Gaussian coordinates of a two-dimensional surface as "nothing other than an arbitrary deformed and stretched planar Cartesian coordinate system" [Einstein 1920, 145], which are straightforwardly applied to the case of general relativity as the Gaussian coordinates of a four-dimensional space-time. (4) In the persistence, in the context of general relativity, of the reference to the effect of the gravitational field on rods and clocks, e.g., in redshift derivations, which, in cases (1) and (2), clearly gives rise to a perverted geodesy. Section 4 is a complementary section in which we address some further elements pointing to the presence of a perverted geodesy in Einstein's work. We mention briefly the evidence for the presence of a perverted geodesy in Einstein's early metric theory of gravitation (the Entwurf theory) and in Einstein's work on Nordström's scalar theory of gravitation. In the final section, we reiterate our thesis, and address some possible criticism and alternatives to it.

2 Perverted geodesy in the rotating disk gravitational redshift derivation

The physical meaning of coordinates in general relativity is not as direct as in special relativity. In the latter case, we identify the reading of unit-measuring rods and clocks with the differential $dx(dy, dz)$ and dt of the line element $ds^2 = -dx^2 - dy^2 - dz^2 + c^2 dt^2$. In the case of general relativity this expression is only valid locally and in general $ds^2 = \Sigma_{\mu\nu} g_{\mu\nu} dx_\mu dx_\nu$. The Gaussian coordinates $x_\mu(x_\nu)$ are in this expression related to local measurements made with rods and clocks through the metric $g_{\mu\nu}$. This was already clear to Einstein in his early attempts to produce a metric field theory of gravity based on Riemann geometry and tensor calculus—the Entwurf theory.³ According to Einstein:

The relation between coordinate differentials on the one hand and measuring lengths and times on the other hand is given

3. Einstein worked on a tentative non-covariant metric field theory, the so-called Entwurf theory, from mid-1912 to late 1915, when he discovered the field equations of general relativity [see, for instance Norton 2005].

in this way; since the quantities $g_{\mu\nu}$ enter into this relationship, the coordinates by themselves have no physical meaning. [Einstein 1913, 211]

Einstein gives a detailed introduction to Gaussian coordinates in his early book on special and general relativity, which was published in 1917. We begin with the original context in which Gauss developed this type of coordinates: its application to curved surfaces. Einstein asks us to imagine “a system of arbitrary curves drawn on the surface of the table” [Einstein 1917, 340]. We have a mesh of “horizontal” and “vertical” curves u and v , and each point of the surface is identified by the intersection of two of these curves. It follows that “a value of u and a value of v belong to every point on the surface” [Einstein 1917, 341]. These numbers are our Gaussian coordinates. Accordingly, “two neighbouring points P and P' on the surface then correspond to the coordinates $P : u, v$ [and] $P' : u + du, v + dv$, where du and dv signify very small numbers” [Einstein 1917, 341]. Here we do not have any notion of length or distance between P and P' . For that purpose, we can do as follows:

We may indicate the distance (line-interval) between P and P' , as measured with a little rod, by means of the very small number ds . Then according to Gauss we have in the latter case $ds^2 = g_{11}du^2 + 2g_{12}dudv + g_{22}dv^2$. [Einstein 1917, 341]

We see that the Gaussian coordinates do not have a direct metrical significance. As Einstein remarks, “the Gaussian method can be applied also to a continuum of three, four or more dimensions” [Einstein 1917, 342]. Accordingly:

We refer the four-dimensional space-time continuum in an arbitrary manner to Gauss coordinates. We assign to every point of the continuum (event) four numbers x_1, x_2, x_3, x_4 (coordinates), which have not the least direct physical significance. [Einstein 1917, 349]

For this approach to be meaningful, it is crucial that our measuring rods (our “little rods”) and clocks correspond to standards of length and time that do not change depending on where and when the measurements are being made. Accordingly, it is an assumption of the theory that the rate of clocks (or the length of rods) is independent from their path of transfer.⁴ According to Einstein, it is this assumption that enables an invariant ds , as we can see, e.g., in a letter Einstein wrote to Weyl in 1918:

4. See, for instance, [Einstein 1921b, 225], [Synge 1960, 106], [Geroch 1972, 8]. While Einstein considers rods and clocks equally, in Malament, Synge, or Geroch’s approaches, for instance, only clocks have a special role; light rays “complement” clocks to provide a measurement of spatial lengths [Malament 2012, 118], [Synge 1960, 112–113], [Geroch 1972, 11–12]. To present and analyze Einstein’s ideas more easily, we will adopt his original views and mention rods and clocks. However, we can define a practically-rigid rod by what Darrigol called “chrono-optical control of

If light rays were the only means of establishing empirically the metric conditions in the vicinity of a space-time point, a factor would indeed remain undefined in the distance ds (as well as in the $g_{\mu\nu}$'s). This indefiniteness would not exist, however, if the measurement results gained from (infinitesimal) rigid bodies (measuring rods) and clocks are used in the definition of ds . A timelike ds can then be measured directly through a standard clock whose world line contains ds .

Such a definition for the elementary distance ds would only become illusory if the concepts “standard measuring rod” and “standard clock” were based on a principally false assumption; this would be the case if the length of a standard measuring rod (or the rate of a standard clock) depended on its prehistory. If this really were the case in nature, then no chemical elements with spectral lines of a specific frequency could exist, but rather the relative frequencies of two (spatially adjacent) atoms of the same sort would, in general, have to differ. [CPAE, vol. 8, 533]

An immediate consequence of this is that the standard (or unit-measuring) rods and the standard (or unit-measuring) clocks cannot be considered to be affected by the gravitational field. If we have a unit clock in a particular region of the gravitational field (i.e., of space-time) and move it to another region, it is still a unit clock—that is, its rate is not affected by the path of transfer *or* by its new location in space-time.⁵

In his detailed analysis of Einstein's review paper on general relativity from 1916, Darrigol noticed its “lack of a clear distinction between heuristic and deductive arguments” [Darrigol 2015, 163]. This can be seen, for instance, in

rigidity” [Darrigol 2015, 167]. This implies that, from a modern perspective, we can justify adopting rods and clocks on an equal footing. As Darrigol says, “what is truly incompatible with general relativity is the existence of extended rigid bodies” [Darrigol 2015, 167].

There is also a more “technical” detail at play. The local Minkowskian space-time corresponds to a local inertial reference frame, which we can regard as being in free fall in the gravitational field, see, for instance [Einstein 1922, 322–323], [Einstein 1923, 78]. If we consider, for instance, a clock “at rest” in a gravitational field, we must check that its rate is not affected by the non-gravitational forces that “pull” the clock away from its free fall along a geodesic. We will hold that, when mentioning measurements made by rods and clocks, these are part of a free falling local inertial frame, or that these measurements would be identical to that of rods and clocks in free fall momentarily at rest in relation to the rods and clocks being used to make the measurements.

5. In this presentation of the “standard interpretation” of general relativity in terms of a concrete geodesy, we make reference to a number of texts by Einstein. However, as we will see, he did not offer any consistent presentation of this view in his writings. (Our references are to [Einstein 1921a,b], [Einstein 1922, 323], and some correspondence, including [CPAE, vol. 8, 529, 533].) Most elements we found can be interpreted as referring to a perverted geodesy.

Einstein's derivation of the gravitational redshift. In his 1916 paper, Einstein made a (heuristic) derivation of the redshift in which a clock was taken to be affected by the gravitational field, and the "temporal" Gaussian coordinate was given a direct metrical significance as corresponding to the measured time interval of the clock "at rest" in the gravitational field [Einstein 1916a, 197–198], [Darrigol 2015, 172–173].⁶ Darrigol considered two possible explanations for this state of affairs, one of them being that "Einstein seems to be starting from a fictitious Minkowskian space-time and to be treating the departure from Minkowskian geometry as an effect of the gravitational field on the rate of clocks" [Darrigol 2015, 172].⁷ Darrigol called this view "perverted geodesy".

For the purpose of this paper it will be useful to consider the redshift derivation made by Einstein in his 1917 book, which relied on the equivalence principle and clocks located in a rotating disk.⁸ Einstein considers a disk rotating with a constant angular velocity ω (relative to an inertial reference frame K), which constitutes an accelerated reference frame K' . Einstein considers a clock located at a distance r from the center of the disk. According to special relativity, the frequency of the clock (number of ticks of the clock per unit time) is $v = v_0(1 - \frac{\omega^2 r^2}{2c^2})$, where v_0 is the frequency of an identical clock at rest at the origin. By resort to the equivalence principle, judged from K' the clock "is in a gravitational field of potential Φ ", where $\Phi = -\frac{\omega^2 r^2}{2}$ [Einstein 1917, 389]. From this result—taken to "hold quite generally", and regarding "an atom which is emitting spectral lines as a clock"—Einstein concludes:

6. This general relativistic derivation is basically the same as the derivations made using the Entwurf theory [Einstein 1913, 216–218], [Einstein 1914, 82].

7. Regarding this derivation, Earman & Glymour have already noted that "the derivation actually relied on the same ideas as the 1907 and 1911 heuristic derivations, especially the idea that the rate of clocks is affected by the gravitational field" [Earman & Glymour 1980, 182]. Overall, regarding Einstein's and others heuristic derivations, Earman & Glymour claim that "all heuristic derivations of the red shift can be faulted on various technical arguments. But to raise such objections is to miss the purpose of heuristic arguments, which is not to provide logically seamless proofs but rather to give a feel for the underlying physical mechanism. It is precisely here that most of the heuristic red shift fail—they are not good heuristics" [Earman & Glymour 1980, 181].

8. Even if this "derivation" appeared in a "popular" exposition of the theory from 1917, we must bear in mind that Einstein did not consider it less valuable than his heuristic field theory derivations. We can see this in his reliance on this derivation in an unpublished 1920 manuscript, intended for publication in *Nature* [Einstein 1920, 141–142], and in a letter to Eddington from the previous year, where Einstein claimed that this derivation showed the gravitational redshift "is an absolutely compelling consequence of relativity theory" [CPAE, vol. 9, 184]. It is clear that Einstein was confident of this derivation. Importantly, in this letter he also remarks that "measuring rods and clocks exhibit a behavior independent of their prehistories" [CPAE, vol. 9, 184]. It seems that Einstein did not notice the incompatibility between the presuppositions and consequences implied in the derivation and the physical assumption of "independence from past history" regarding rods and clocks.

An atom absorbs or emits light of a frequency which is dependent on the potential of the gravitational field in which it is situated.

The frequency of an atom situated on the surface of a heavenly body will be somewhat less than the frequency of an atom of the same element which is situated in free space (or on the surface of a smaller celestial body). [...] Thus a displacement towards the red ought to take place for spectral lines produced at the surface of stars as compared with the spectral lines of the same element produced at the surface of the earth. [Einstein 1917, 389–390]

In this derivation of the gravitational redshift, we have an “underlying physical mechanism” that leads to the gravitational redshift and that is the effect of the gravitational field on clocks. According to this derivation, clocks go slower in the vicinity of a gravitational source. (In this case the “gravitational field” is stronger the further away the clock is from the center of the disk.)⁹ This view is in complete contradiction with the physical assumption of the “independence from past history” of unit-measuring rods and clocks that sustains and gives physical meaning to the invariant line element ds .

Looking more closely into this derivation we can also see a perverted geodesy at work. In fact, the perverted geodesy goes hand in hand with the idea that clocks and rods are affected by the gravitational field. The derivation shows us that the frequency of a clock in a rotating disk changes according to the clock's distance to the center, according to the expression $v = v_0(1 - \frac{\omega^2 r^2}{c^2})$. We might say that we have a curved time. (We will see in the next section that there is an equivalent “effect” regarding rods in a rotating disk, so that instead of a Euclidean space we have a non-Euclidean space. Overall, we have a curved space-time.) As Einstein recognizes for the similar case of rods in a rotating disk:

Throughout this [derivation] we have to use the Galilean (non-rotating) system K as reference-body, since we may only assume the validity of the results of the special theory of relativity relative to K (relative to K' a gravitational field prevails). [Einstein 1917, 334]

The mathematical expression $v = v_0(1 - \frac{\omega^2 r^2}{c^2})$ is derived in relation to K —i.e., in relation to an underlying non-curved time. The special relativistic time dilation, as determined in K , implies in K that the clock appears to run slower the more distant it is from the center. According to Einstein this result also holds in K' , being reinterpreted, applying the equivalence principle, in terms of the presence of a gravitational field—i.e., we have $v = v_0(\frac{1-\Phi}{c^2})$, where

9. This “underlying physical mechanism” is already present in Einstein's initial derivations in terms of the application of the equivalence principle in the case of a uniformly accelerated reference frame [Einstein 1907, 302–307], [Einstein 1911, 384–385].

$\Phi = \frac{-\omega^2 \gamma^2}{2}$ [Einstein 1917, 389]. In this way, it is the departure of the clock's rate from its underlying non-curved/uniform time due to the gravitational field (the rotation of the disk) that curves time. We can say, using a phrasing similar to Darrigol's, that time is originally non-curved but it appears to be curved when clocks are affected by a gravitational field: we start with the non-curved time of a Minkowskian space-time, as described in an inertial reference frame K , and we treat the departure from the Minkowskian non-curved time as an effect of the gravitational field on the rate of clocks. We have an instance of a perverted geodesy.

3 Perverted geodesy in Einstein's practical geometry

While Einstein's mature views on geometry appeared in his famous 1921 lecture, "Geometry and experience" [Einstein 1921a], some of his views had already been "consolidated" and published in a paper on the Entwurf theory in late 1914. We start with the idea of geometry as "pure" mathematics:

[Euclidean] geometry means originally only the essence of conclusions from geometric axioms; in this regard it has no physical content. [Einstein 1914, 78]

Then comes the view that geometry can become a physical science:

[Euclidean] geometry becomes a physical science by adding the statement that two points of a "rigid" body shall have a distinct distance from each other that is independent of the position of the body. [Einstein 1914, 78]¹⁰

(By 1921 this view is elaborated in a way that only geometry as a physical science is meaningful in the present stage of development of physics.) This leads to a view that Einstein will maintain throughout:

After this amendment, the theorems of this amended [Euclidean] geometry are (in a physical sense) either factually true or not true. [Einstein 1914, 78]

That is, we can experimentally determine if the adopted geometry is in agreement with observation and/or experimental results. Regarding Euclidean geometry, Einstein's position is, therefore, that "Euclidean geometry too—as it is used in physics—consists of physical theorems that, from a physical

10. As early as 1912, Einstein remarked that "the propositions of Euclidean geometry acquire a physical content through our assumption that there exist objects that possess the properties of the basic structures of Euclidean geometry" [Einstein 1912, 28].

aspect, are on an equal footing with the integral laws of Newtonian mechanics" [Einstein 1914, 78].

Einstein mentions Euclidean geometry as a physical science briefly once more in his 1916 review paper on general relativity. Here, Einstein is more explicit regarding what becomes of Euclidean geometry as a physical science: "the laws of [Euclidean] geometry, even according to the special theory of relativity, are to be interpreted directly as laws relating to the possible relative positions of solid bodies at rest" [Einstein 1916a, 148]. Here we find expressed (implicitly) the view that the amended or physical Euclidean geometry only applies for solid bodies in inertial motion. In fact, a few pages later, Einstein considers a coordinate system K' in uniform rotation around the Z -axis of an inertial (or Galilean) system of reference K , both of them having the same origin. Einstein concludes that "Euclidean geometry does not apply to K' " [Einstein 1916a, 152]. To arrive at this conclusion, he considers "a circle around the origin in the X, Y plane of K [which] may at the same time be regarded as a circle in the X', Y' plane of K' " [Einstein 1916a, 152]. If we measure the circumference and diameter of this circle using unit-measuring rods at rest relative to K , the quotient is π . However, if we make the same measurements with unit-measuring rods at rest relatively to K' , the quotient is greater than π . According to Einstein:

This is readily understood if we envisage the whole process of measuring from the "stationary" system K , and take into consideration that the measuring-rod applied to the periphery undergoes a Lorentzian contraction, while the one applied along the radius does not. Hence Euclidean geometry does not apply to K' . [Einstein 1916a, 152]

As with the redshift derivation considered in the previous section, we have a case of a perverted geodesy. Applying the equivalence principle, we might consider the gravitational field "existing" in K' to affect the length of the unit-measuring rods located on the periphery of the circle in a way similar to its effect on clocks. Again, we start with a description in a Minkowskian space-time and, by calculations made in relation to the inertial reference frame K , we determine the "distortion" suffered by unit-measuring rods, which leads to conclude from "measurements" made with the whole set of rods (the ones along the radius and the ones along the periphery) that space is non-Euclidean.

After considering the issue of spatial geometry, Einstein discusses what happens to clocks rotating with K' . We have a situation similar to that of the unit-measuring rods along the periphery. Due to the time dilation, a "clock at the circumference—judged from K —goes more slowly than [a clock at the origin]" [Einstein 1916a, 152]. In this case he does not mention the gravitational redshift, which, however, is a direct consequence. As mentioned in the previous section, this is another case of a perverted geodesy.

Taking together the results for rods and clocks in a rotating coordinate system (which we associate with a rotating reference frame, the disk), we

see that, in this case, rather than “starting from a fictitious Minkowskian space-time” [Darrigol 2015, 172], we start from what we might call a “background” Minkowskian space-time (which is always “there” for the coordinate system K), and due to the “influence” of the gravitational field (i.e., the rotation) we have in K' an “effective” curved space-time.

Einstein gives a new presentation of his views on geometry in his 1917 book. Again, Einstein starts with “pure” geometry. As such, “geometry, however, is not concerned with the relation of the ideas involved in it [e.g., points or straight lines] to objects of experience, but only with the logical connection of these ideas among themselves” [Einstein 1917, 250]. As in 1914, we can consider geometry to be amended so that we can use it in physics. At this point Einstein still treats geometry at two levels. It is not clear that the amended geometry is necessary except for practical reasons:

We now supplement the propositions of Euclidean geometry by the single proposition that two points on a practically rigid body always correspond to the same distance (line-interval), independently of any changes in position to which we may subject the body. [Einstein 1917, 251]

The wording here differs from that of the statement made in late 1914. As in early 1916, Einstein stresses that “the propositions of Euclidean geometry then resolve themselves into propositions on the possible relative position of practically rigid bodies” [Einstein 1917, 251]. After this sentence, Einstein presents two of the key ideas already encountered in 1914:

Geometry which has been supplemented in this way is then to be treated as a branch of physics. We can now legitimately ask as to the “truth” of geometrical propositions interpreted in this way, since we are justified in asking whether these propositions are satisfied for those real things we have associated with the geometrical ideas. [Einstein 1917, 251]

After this, and similarly to the 1916 paper, Einstein addresses the case of a reference frame K' rotating uniformly in relation to an inertial reference frame K . Our rotating frame consists of a plane circular disk rotating about its center. Let us consider two identical clocks, one at the center of the disk and the other at its periphery. Applying the results of special relativity to the clock located at the periphery with a constant angular velocity (in K), we find that this clock “goes at a rate permanently slower than that of the clock at the center of the circular disk” [Einstein 1917, 333]. Einstein extends this result, arguing that “in every gravitational field, a clock will go more quickly or less quickly, according to the position in which the clock is situated (at rest)” [Einstein 1917, 333]. In the case of spatial geometry, we imagine that an “observer applies his standard measuring-rod [...] tangentially to the edge of the disk” [Einstein 1917, 333]. Again, applying special relativity, Einstein concludes that:

As judged from the Galilean system, the length of this rod will be less than 1 [...] on the other hand, the measuring rod will not experience a shortening in length, as judged from K , if it is applied to the disk in the direction of the radius. If then, the observer first measures the circumference of the disk with his measuring-rod and then the diameter of the disk, on dividing the one by the other, he will not obtain as quotient the familiar number $\pi = 3.14\dots$, but a larger number, whereas of course, for a disk which is at rest with respect to K , this operation would yield π exactly. [Einstein 1917, 334]

From this result, Einstein concludes that “the propositions of Euclidean geometry cannot hold exactly on the rotating disk, nor in general in a gravitational field, at least if we attribute the length 1 to the rod in all positions and in every orientation” [Einstein 1917, 334]. It is important to notice that Einstein’s conclusion is dependent on accepting that the measuring rod is always a unit rod. Here Einstein links the experimental determination of what geometry we have to a previous stipulation of the length of our measuring rod (and its independence from orientation). If an observer in K' could choose the rods at the periphery to have a length different from one, taking into account the Lorentz contraction, these rods could be considered, in K , as having the length 1, and we could retain a Euclidean geometry. However, these rods would have, in K' , a length different from that of the rods positioned along the radius—that is, the lengths of the rods in K' would be taken to depend on the orientation.

The next chapter of Einstein’s book is a preparation for the subsequent introduction of the notion of Gaussian coordinates. This chapter is informed by Einstein’s analysis of the rotating disk. Einstein asks us to imagine the disposition (placement) of little rods of equal length (i.e., standard/unit measuring rods) on the surface of a marble table. We lay these rods out in quadrilateral figures, thereby making a network of squares on the surface of the marble table. According to Einstein, “if everything has really gone smoothly, then I say that the points of the marble slab constitute a Euclidean continuum with respect to the little rod, which has been used as a ‘distance’ (line-element)” [Einstein 1917, 337]. Again, it is essential to take into account what the measurement standard is in relation to which we are determining the geometry.

Now we imagine that “we heat the central part of the marble slab, but not the periphery” [Einstein 1917, 338]. Here we can make an analogy with the case of the rotating disk, where the central part is at rest and, as we approach the periphery, the angular velocity increases (in this case the temperature is falling). According to Einstein, in this case, “two of our little rods can still be brought into coincidence at every position on the table” [Einstein 1917, 338]. This means that the rods are congruent independently of the position at the table—i.e., independently of the temperature. They are all affected in

the same way by the temperature, just as the rods at the periphery of the rotating disk all suffer the same Lorentz contraction, as determined in the inertial reference frame. What happens is that “our construction of squares must necessarily come into disorder during the heating, because the little rods on the central region of the table expand, whereas those on the outer part do not” [Einstein 1917, 338]. As such, “with reference to our little rods—defined as unit lengths—the marble slab is no longer a Euclidean continuum” [Einstein 1917, 338]. This is exactly the same situation as the rotating disk: if we take the rods at the periphery to be unit rods, we must conclude that the disk does not form a Euclidean continuum. In the case of the marble slab, Einstein gives an example of how we could avoid the effect of the temperature by using different types of rod, ones unaffected by the temperature of the marble slab (or at least not affected in the same way). In this case, “it is possible quite naturally to maintain the point of view that the marble slab is a ‘Euclidean continuum’ ” [Einstein 1917, 338]. But let us imagine a situation in which “rods of every kind (i.e., of every material) were to behave in the same way as regards the influence of temperature” [Einstein 1917, 338]. In this situation, we cannot detect the effect of the temperature on measuring rods experimentally. The same occurs with gravity—or with the case of the rotating disk when applying the equivalence principle. According to Einstein, in this case, “our best plan would be to assign the distance one to two points on the slab, provided that the ends of one of our rods could be made to coincide with these two points” [Einstein 1917, 338–339].

In this example, as in the rotating disk case, we have a perverted geodesy. We have the “background” Euclidean geometry of the marble slab. However, since “we heat the central part of the marble slab, but not the periphery [...] the little rods on the central region of the table expand, whereas those on the outer part do not” [Einstein 1917, 338]. If an “observer” does not have independent means of determining the length expansion of the little rods, this means that he/she cannot determine their “real” length experimentally. By taking the rods to be unit rods (even if we consider that they are “influenced” by the temperature of the marble slab), we must conclude that the geometry of the marble slab, as determined using the rods, is non-Euclidean.

Returning to the “seed” of this work, Darrigol’s footnote, it is important to note that Darrigol does not simply remark that, in his 1917 book, Einstein adopted a perverted geodesy in the case of a marble slab with a temperature gradient. Darrigol argues that Einstein also relies on a perverted geodesy when “he compares the non-Euclidean geometry of general relativity with the apparent geometry of a table top of uneven temperature when gauged with dilatable rods” [Darrigol 2015, 172]. That is, Darrigol is saying that Einstein relies on a perverted geodesy in case of the non-Euclidean geometry of general relativity—i.e., in relation to the curved space-time.

Einstein does not explicitly make this comparison, but when he considers the consequences of the possibility that “rods of every kind (i.e., of every material) [...] behave in the same way as regards the influence of temperature”

[Einstein 1917, 338], he is drawing a clear analogy with the universal “effect” of gravity. And as with the case of the marble slab, where the measuring rods are heated differently depending of their location—i.e., have their “behavior” influenced by the temperature “field” of the marble slab—Einstein speaks, in the context of the four-dimensional space-time of general relativity, of the (physical) “behavior” of measuring rods and clocks “under the influence of the gravitational field” [Einstein 1917, 356]. Darrigol’s view, that Einstein maintained a perverted geodesy also in the case of general relativity, seems plausible. Below, we consider some other “clues” which reinforce this view.

Einstein’s next paper presenting his views on geometry was an unpublished manuscript written in 1920. By this time, he had already developed key ideas which consolidated views he had held since at least 1914, but these are not included in his treatment of geometry in the 1920 paper. During 1918, Einstein felt that it was necessary to make explicit that his theory assumes that the length of a standard measuring rod, or the rate of a standard clock, do not depend on their past history.¹¹ This assumption is strengthened by taking into account that atoms as clocks (which also enable a definition of length) are independent from their past history. According to Einstein, if this assumption was incorrect, “no chemical elements with spectral lines of a specific frequency could exist” [CPAE, vol. 8, 533].

In the 1920 paper, Einstein refers to the independence of rods and clocks from their past history solely in the context of special relativity, substituting his earlier reference to the more particular case of the boostability assumption [Einstein 1920, 127].¹² He also presents, once again, his rotating disk argument for the adoption of a non-Euclidean geometry (and the gravitational redshift derivation), along the lines of the presentations given in 1916 and 1917 [Einstein 1920, 141–143]. His views on geometry are mostly still those presented in 1914, 1916, and 1917 [Einstein 1920, 144, 146, and 145, respectively]. However, an analogy in the paper presents the Gaussian coordinates of a curved surface in terms of a perverted geodesy. Writing about a Gaussian coordinate system defined over a surface, Einstein remarks that:

[A Gaussian coordinate system] is—speaking graphically—nothing other than an arbitrary deformed and stretched planar Cartesian coordinate system. It is by virtue of this bending that Gaussian coordinates no longer have any physical meaning whatsoever. [Einstein 1920, 145]

We must take into account the fact that, in the heated marble slab example and the rotating disk case, the rods are, so to speak, “deformed and stretched”

11. See, for instance, [Giovannelli 2014], [Ryckman 2005, 85–94].

12. According to Einstein, it is an assumption of special relativity that measuring rods and clocks, when boosted from a state on inertial motion into another, maintain their length and rates [Einstein 1907, 260], [Einstein 1910, 130]; see also [Bacelar Valente 2016].

by the effect of temperature or Lorentz contraction. (Time dilation leads to a similar effect with the clocks in the rotating disk.) If we think of the space and time measurements as made by rods and clocks spread over a material reference frame (the marble slab or the rotating disk), it is their “deformation” in comparison to the “background” flat Minkowskian space-time that leads to a view in terms of a non-Euclidean space-time. While Einstein applies the analogy in the case of a two-dimensional surface, not yet in relation to the four-dimensional space-time of general relativity, we have to bear in mind that the extension of Gaussian coordinates to this later case is straightforward (as we have mentioned in the previous section [see, for instance Einstein 1917, 340–344]). This gives us a further indication that Einstein relies on a perverted geodesy in relation to the space-time geometry of general relativity.

It was only in a 1921 paper that Einstein’s views on geometry previous to 1918 were “integrated” with the views he developed regarding rods and clocks in 1918. He begins “Geometry and experience”, as he does the brief paragraphs of the 1914 paper, with axiomatic geometry, which as such is not able to make “assertions as to the behavior of real objects” [Einstein 1921a, 210]. According to Einstein:

To be able to make such assertions, geometry must be stripped of its merely logical-formal character by the coordination of real objects of experience with the empty conceptual schemata of axiomatic geometry [...]. Then the propositions of Euclid contain affirmations as to the behavior of practically-rigid bodies. [Einstein 1921a, 210–211]

With this amendment of axiomatic geometry, Einstein concludes, much as he did in 1914, that “geometry thus completed is evidently a natural science. [...] We will call this completed geometry ‘practical geometry’” [Einstein 1921a, 211]. There is nevertheless a difference with his previous remarks on geometry. Now we must consider that, prior to coordination with practically-rigid bodies, geometry is still not completed. Once again, as in 1914, completed geometry as a physical science can be submitted to experimentation:

The question whether the practical geometry of the universe is Euclidean or not has a clear meaning, and its answer can only be furnished by experience. [Einstein 1921a, 211]

Einstein then considers two different possible points of view on geometry, something not present in his previous remarks on geometry. In the first, “we reject the relation between the practically-rigid body and geometry” [Einstein 1921a, 212]. It follows, according to Einstein, that:

Geometry (G) predicates nothing about the behavior of real things, but only geometry together with the totality (P) of physical laws can do so. Using symbols, we may say that only the sum of (G) + (P) is subject to experimental verification. Thus

(*G*) may be chosen arbitrarily, and also parts of (*P*); all these laws are conventions. All that is necessary to avoid contradictions is to choose the remainder of (*P*), so that (*G*) and the whole of (*P*) are together in accord with experience. [Einstein 1921a, 212]

The second point of view is that of practical geometry. Why are we entitled to such a point of view? According to Einstein, part of the answer is that:

In the present stage of development of theoretical physics these concepts must still be employed as independent concepts; for we are still far from possessing such certain knowledge of the theoretical principles of atomic structure as to be able to construct solid bodies and clocks theoretically from elementary concepts. [Einstein 1921a, 213]

To put it simply, the first option is not viable because we cannot construct rods and clocks from (*G*) + (*P*). They are irreducible conceptual elements that we coordinate directly with (*G*). This also means that, in the present stage of development of physics, geometry must be completed, rather than simply amended or supplemented as Einstein had previously suggested. But is practical geometry in fact viable? An answer to this question is already present in a 1918 letter from Einstein to Weyl, already discussed in the previous section:

The measurement results gained from (infinitesimal) rigid bodies (measuring rods) and clocks are used in the definition of *ds*. A timelike *ds* can then be measured directly through a standard clock whose world line contains *ds*.

Such a definition for the elementary distance *ds* would only become illusory if the concepts “standard measuring rod” and “standard clock” were based on a principally false assumption; this would be the case if the length of a standard measuring rod (or the rate of a standard clock) depended on its prehistory. If this really were the case in nature, then no chemical elements with spectral lines of a specific frequency could exist, but rather the relative frequencies of two (spatially adjacent) atoms of the same sort would, in general, have to differ. [CPAE, vol. 8, 533]

Einstein presents a similar argument in “Geometry and experience”, in a section of the text which addresses the adoption of practical geometry [Einstein 1921a, 213–214]. From this it should be evident that the unit-measuring rods and clocks are not “influenced” by the gravitational field. This is in strict contradiction with the presupposition, which leads to a perverted geodesy, that rods and clocks are affected by the gravitational field.

Einstein made further remarks on geometry relevant for our current purposes in his lectures on special and general relativity at Princeton in 1921,

which were published as a book the following year. Here, Einstein once again considers a coordinate system K' rotating uniformly relative to an inertial system K [Einstein 1922, 319–320].¹³ Einstein concludes that:

The gravitational field influences and even determines the metrical laws of the space-time continuum. If the laws of configuration of ideal rigid bodies are to be expressed geometrically, then in the presence of a gravitational field the geometry is not Euclidean. [Einstein 1922, 321]

With an almost implicit reference to perverted geodesy, Einstein says that, if the (physical) laws of chronometry and spatial geometry as determined by the time dilation and Lorentz contraction are expressed geometrically, we arrive at a curved space-time. Taking his previously published remarks into account, this means that the rods and clocks are implicitly taken to be unit rods and unit clocks. Symbolically, we go from $(G_{\text{Euclidean}}) + (P_{\text{Lorentz contracted rods + clocks with time dilation}})$ to $(G_{\text{non-Euclidean}}) + (P_{\text{unit rods + unit clocks}})$. In the back of Einstein's mind, there seems to be the idea that non-Euclidean geometry (i.e., curved space-time) “expresses geometrically” the effect of the gravitational field on rods and clocks, which are now treated as unit rods and unit clocks. In fact, Einstein maintains the view in these lectures, in the context of general relativity, that rods and clocks are affected by the gravitational field [Einstein 1922, 351–352]. This is particularly clear in the case of clocks (giving rise to the gravitational redshift), in relation to which Einstein argues that:

The interval between two beats of the unit clock ($dT = 1$) corresponds to the “time” $1 + \frac{k}{8\pi} \int \frac{\sigma dV_0}{r}$ in the unit used in our system of coordinates. The rate of a clock is accordingly slower the greater is the mass of the ponderable matter in its neighbourhood. [Einstein 1922, 352]

13. Let us imagine that we have identical measuring rods “laid in series along the periphery and the diameter of the circle, at rest relative to K' ” [Einstein 1922, 319]. Let U be the number of rods along the periphery, and D the number of rods along the diameter. If K' does not rotate relative to K , $\frac{U}{D} = \pi$. As we have already seen, if K' is rotating, the rods along the periphery experience a Lorentz contraction but not the rods along the diameter. In this case, $\frac{U}{D} > \pi$. According to Einstein, “it therefore follows that the laws of configuration of rigid bodies with respect to K' do not agree with the laws of configuration of rigid bodies that are in accordance with Euclidean geometry” [Einstein 1922, 320]. As we have seen, when placing two identical clocks, one at the center of the circle and the other at the periphery (rotating with K'), due to the time dilation “the clock on the periphery will go slower than the clock at the center” [Einstein 1922, 320]. Taking into account the principle of equivalence, “ K' may also be considered as a system at rest, with respect to which there is a gravitational field” [Einstein 1922, 320].

This seem to us to offer further hints that Einstein's views on geometry (in the context of general relativity) were permeated by a perverted geodesy.¹⁴

4 Further evidence for a perverted geodesy in Einstein's work

As we have seen, Einstein's views on geometry after the development of the Entwurf theory seem to be permeated by a perverted geodesy. But why would the adoption of Riemann geometry and tensor calculus be accompanied by this view? An explanation may lie in the crucial role the rotating disk argument had in the adoption of the new mathematics. According to Stachel, the rotating disk example seems to be "a 'missing link' in the chain of reasoning that led [Einstein] to the crucial idea that a nonflat metric was needed for a relativistic treatment of the gravitational field" [Stachel 1986, 245]. As Stachel discovered, Einstein wrote in his private correspondence that the argument for a non-Euclidean geometry in the case of a rotating disk was central to his decision to adopt Riemann geometry in his quest for a theory of gravitation. In his published writings, however, Einstein did not mention this important point [Stachel 1986, 252–253].

On a number of occasions, Einstein put forward a compact argument for the adoption of the general expression for the line element and a generalization of the flat space-time equations $R^\mu_{\nu\sigma\tau} = 0$. This argument is based on the equivalence principle, and we can detect within it a "trace" of the rotating disk argument. It goes as follows: since the interval of the Minkowski space-time element $ds^2 = -dx^2 - dy^2 - dz^2 + c^2 dt^2$ can be given by $ds^2 = \Sigma_{\mu\nu} g_{\mu\nu} dx_\mu dx_\nu$ in an arbitrary coordinate system, and due to the equivalence principle it describes a gravitational field, then the "free-field" Minkowski space-time can be interpreted as a "special case" satisfying the "Riemann condition" $R^\mu_{\nu\sigma\tau} = 0$ to be generalized for an arbitrary gravitational field [Einstein 1936, 308–309], [Einstein 1954, 372–375]. This more abstract compact argument can be related very directly to the heuristic rotating disk argument. The "free-field" Minkowski space-time line element $ds^2 = -dx^2 - dy^2 - dz^2 + c^2 dt^2$ can be described in terms of a rotating frame of reference, e.g., as $ds^2 = -dr'^2 - r'^2 d\phi'^2 - dz'^2 - 2\omega r' d\phi' dt' + (c^2 - r'^2 \omega^2) dt'^2$ (see, for instance [Dieks 2004, 31–32]). This is a particular case of the general expression $ds^2 = \Sigma_{\mu\nu} g_{\mu\nu} dx_\mu dx_\nu$ in an arbitrary coordinate system. Applying

14. As late as 1938, Einstein relied on the rotating disk argument to make the case for a non-Euclidean geometry in the presence of a gravitational field. We still have the "underlying physical mechanism" of the effect of the gravitational field on rods and clocks, which would explain the gravitational redshift. The presence of this "mechanism" leads to a perverted geodesy, at least in this case. Einstein also returns to the example of the marble slab, again an example of a perverted geodesy [Einstein & Infeld 1938, 238–255].

the equivalence principle, we can reinterpret these expressions as resulting from, and expressing the presence of, a gravitational field; by loosening its application beyond the case of the flat Minkowski space-time, we consider more general kinds of space-times and gravitational fields—that is, we go from $R^\mu_{\nu\sigma\tau} = 0$ to $R_{\nu\sigma} = R^\mu_{\nu\sigma\mu}$ (see, for instance [Norton 1985, 230–232]).

Considered in this way, the perverted geodesy of the heuristic rotating disk argument appears compatible with the new mathematical formalism, in particular with the curved space-time. We can see this by considering some of Einstein’s work on the Entwurf theory and comparisons with Nordström’s gravitation theory.

In a paper comparing both theories, Einstein calculates the Newtonian approximation in his Entwurf theory [Einstein 1913, 216–219]. We find another example of this approximation, in the context of the Entwurf theory, in “The formal foundations of the general theory of relativity” [Einstein 1914]; it is basically the one he made in his 1916 review paper on general relativity. As Darrigol points out, this procedure “has the defect of presuming a partial metric interpretation of the coordinates before the metric is given” [Darrigol 2015, 171]. Darrigol notes that this might be due to Einstein’s reliance on a perverted geodesy [Darrigol 2015, 172].¹⁵ We can see the plausibility of this from the results Einstein obtained when adopting the Newtonian approximation. According to Einstein, “the rate of a clock depends on the gravitational potential [...]. Thus, the greater the masses arrayed in its vicinity, the slower the clock runs” [Einstein 1913, 218]. In this way, the change in the time coordinate in relation to the Minkowskian case arises due to the effect of the gravitational field: we have a perverted geodesy.¹⁶

Darrigol speaks of Einstein’s use of a fictitious flat space-time [Darrigol 2015, 172]. However, considering the influence Einstein’s work on Nordström’s theory potentially had on Einstein, it might well be the case that Einstein, at some point, thought in terms of an “actual” background flat space-time. (This seems to be the case in some of his comments on geometry, as we saw in the

15. In fact, even in the thirties, Einstein adopted a post-Newtonian approximation in his work on the problem of motion in general relativity [Einstein, Infeld *et al.* 1938], see also [Deruelle & Uzan 2014, 530–532]. In 1915, Einstein adopted a post-Newtonian approximation to calculate the anomaly in the advance of Mercury’s perihelion [Einstein 1915], see also [Eisenstaedt 2002, 258–259]. This reliance, in practice, on a sort of background Minkowski space-time in which the coordinates, in Darrigol’s words, had a “partial metric interpretation” [Darrigol 2015, 171] might indicate that a perverted geodesy was still present in the thirties (see also footnote 14). We must note that this kind of approach is not an “exotic” early scheme adopted by Einstein, see, e.g., [Deruelle & Uzan 2014, 526–538], [Poisson & Will 2014, 290–619]. In this article, we will not treat in any detail the evidence for perverted geodesy arising from Einstein’s perturbative calculations. This subject needs a full-length paper.

16. In a similar calculation made within the Entwurf theory using the Newtonian approximation, Einstein concluded again that “the clock rate [...] increases with the gravitational potential” [Einstein 1914, 82].

previous section.) Briefly, Nordström proposed a scalar theory of gravitation which was Lorentz invariant—i.e., a theory formulated in a Minkowski space-time [Nordström 1912], [Norton 1992, 35–39]. After an important debate, and contributions by Einstein and Laue, Nordström formulated his “second” theory [Nordström 1913], [Norton 1992, 61–76]. In this theory, the length of rods and the rate of clocks were affected by the gravitational field much as Einstein had proposed in the Entwurf theory (see, for instance, [Einstein 1913], [Norton 1992, 65–69]). This means that, while the theory is formulated with an “actual” background Minkowski space-time, the effect of the gravitational field on rods and clocks makes it an effective curved space-time. This is made particularly clear when Einstein and Fokker developed Nordström’s theory directly in terms of Riemann geometry and tensor calculus, showing that it could be seen as a more restricted case of the Entwurf theory [Einstein & Fokker 1914], [Pais 1982, 232–237], [Norton 1992, 76–79]. As Norton remarked:

Under continued pressure from Einstein, Nordström made his theory compatible with the equality of inertial and gravitational mass by assuming that rods altered their length and clocks their rate upon falling into a gravitational field so that the background Minkowski space-time had become inaccessible to direct measurement. As Einstein and Fokker showed in early 1914, the space-time actually revealed by direct clock and rod measurement had become curved, much like the space-times of Einstein’s own theory. [Norton 1993, 5], see also [Deruelle & Uzan 2014, 275]

This result reinforces the view that the curved space-time was a result of the effect of gravity on rods and clocks “existing” in a flat space-time—that is, it reinforces a perverted geodesy view. It is therefore very plausible that the early development of general relativity occurred in the context of a perverted geodesy.¹⁷ Recall from the previous section that Einstein only made the assumption of the independence of rods and clocks from their past history explicit in 1918, in private correspondence (for details, see [Giovannelli 2014]). This is related to the adoption of a concrete geodesy, but had little impact on his subsequent texts on general relativity, since, as we have seen, he did not “retract” his perverted geodesy views.

In connection to this, we must also note that Einstein did not take the state of motion of the local inertial reference frame—in relation to which special relativity is valid—into account more consistently until later [Einstein 1922, 322–323, 350], see also [Einstein 1923, 78]. This may have prevented Einstein associating a natural measure (in terms of the proper time) with

17. The brief treatment of Nordström’s theory and the reference to the post-Newtonian approximation in footnote 15 was motivated by one of the reviewers, who mentioned, in relation to perverted geodesy, the “perturbative methods which Einstein used and which are still used today” and “the influence of Nordström theory on Einstein’s point of view”.

clocks in a gravitational field—because, for instance in his calculations of the redshift in the Entwurf theory, Einstein considered clocks at rest in the gravitational field.¹⁸ But even when he explicitly takes this into account in his Princeton lectures—where he also mentions the independence from past history assumption—Einstein still considers that “the rate of a clock is accordingly slower the greater is the mass of the ponderable matter in its neighbourhood” [Einstein 1922, 352]. The lack of a full development of these ideas in the early days of his metrical field theories might also be part of the reason Einstein did not notice that the Riemannian space-time, which is locally Minkowskian and in which coordinates have no direct physical meaning, imposes, so to speak, a concrete geodesy. Whatever the causes, it seems that from the start Einstein’s efforts to develop a metrical field theory were permeated by a perverted geodesy. We can see this in the following excerpt from the first paper on the Entwurf theory:

From the foregoing, one can already infer that there cannot exist relationships between the space-time coordinates x_1, x_2, x_3, x_4 and the results of measurements obtainable by means of measuring rods and clocks that would be as simple as those in the old relativity theory [...]. We note in this connection that ds is to be conceived as the invariant measure of the distance between two infinitely close space-time points. For that reason, ds must also possess a physical meaning that is independent of the chosen reference system. We will assume that ds is the “naturally measured” distance between the two space-time points [...]. The immediate vicinity of the point (x_1, x_2, x_3, x_4) with respect to the coordinate system is determined by the infinitesimal variables dx_1, dx_2, dx_3, dx_4 . We assume that, in their place, new variables $d\xi_1, d\xi_2, d\xi_3, d\xi_4$ are introduced by means of a linear transformation in such a way that $ds^2 = d\xi_1^2 + d\xi_2^2 + d\xi_3^2 - d\xi_4^2 \dots$ the ordinary theory of relativity holds in this elementary $d\xi$ system. [...] ds is the square of the four-dimensional distance between two infinitely close space-time points [...] [measured] by means of unit measuring rods and clocks [...]. From this one sees that, for given dx_1, dx_2, dx_3, dx_4 , the natural distance that corresponds to these differentials can be determined only if one knows the quantities $g_{\mu\nu}$ that determine the gravitational field. This can also be expressed in the following way: the gravitational field influences the measuring bodies and clocks in a determinate manner. [Einstein & Grossmann 1913, 156–157]

18. See [Einstein 1913, 218], [Einstein 1914, 82]. While in his review paper from 1916 Einstein acknowledges that “we must choose the acceleration of the infinitely small (‘local’) system of coordinates so that no gravitational field occurs” [Einstein 1916a, 154], he still considers a clock “arranged to be at rest in a static gravitational field” [Einstein 1916a, 197].

As it is, that space-time is locally Minkowskian, that space-time coordinates do not have a direct physical meaning, and that ds is an invariant that can be determined by the rods and clocks of a local inertial reference frame, do not have to be seen as implying Einstein's adoption of a concrete geodesy.¹⁹

5 Coda

In this article, we have made the case that Darrigol's claim that Einstein relied on a perverted geodesy is quite plausible, by showing in section 3 that Einstein's views on geometry are permeated with references to perverted geodesy. In section 4, we have considered some further evidence for this in Einstein's early work. With this we are not saying that explanations not relying on a perverted geodesy reading of Einstein's work are to be excluded. At first, it might seem we are presented with a dichotomy: either (1) Einstein believed in a perverted geodesy, or (2) Einstein believed in the concrete geodesy and used the perverted geodesy at most as a heuristic tool (by considering the "effect" of the gravitational field on rods and clocks). Before considering (2), we must note that we do not adopt (1) here. From our point of view, there is no reason to hold that Einstein had a clear "belief" in any distinct position. The presence of inconsistent elements in his work and views seem to us as an indication of this. Our position is that the elements mentioned by Darrigol, complemented with those presented here, make it very plausible that Einstein's views on geometry were permeated by a perverted geodesy. But this does not mean that Einstein was adhering consistently to a perverted geodesy instead of a concrete geodesy.

(2) would avoid the awkwardness of accepting that part of Einstein's work was inconsistently permeated with a perverted geodesy, but we would face the equally awkward task of explaining how Einstein made such inconsistent claims (and derivations) in so much of his work, even if we consider them as the result of a "heuristic tool".²⁰ In arguing for (2), we might try to consider Einstein's references and calculations related to a perverted geodesy as "occasional deviations" from the standard interpretation in terms of a concrete geodesy. But this standard interpretation is nowhere to be found in Einstein's presentation of general relativity. We have a non-mathematical presentation of practical geometry, in terms of what we now identify as a concrete geodesy, in "Geometry and experience", and not before. The only example in this period of a scientific text where concrete geodesy is implicitly

19. This last paragraph addresses worries set forward by another reviewer who defends a view different from our own. We will consider it in the next section where we address views proposed by this reviewer.

20. In this discussion of (2), we will hold that the only element giving rise to a perverted geodesy is the effect of gravitational fields on rods and clocks, and disregard the fact that the Newtonian approximation might also be related to a perverted geodesy, as Darrigol mentions. See also footnote 15.

adopted is in a little work where Einstein develops a theory along the lines of Weyl's theory of gravitation and electromagnetism [Einstein 1921b]. The other case we have found that might relate to a concrete geodesy is a brief reference, in his Princeton lectures (published in 1922), in relation to the invariance of ds , which is similar to Einstein's approach in his correspondence with Weyl in 1918. However, as we have seen, there are several elements in this text which refer to a perverted geodesy. As we have seen in sections 3 and 4, we find (direct or indirect) references to perverted, rather than concrete, geodesy in a large number of Einstein's texts on gravitation, whether these are scientific, semi-popular, or didactic.

There is another view, more nuanced than (2), seemingly difficult to distinguish from the one defended here on the basis of textual evidence. In his paper, Darrigol claims that Einstein's reliance on a perverted geodesy has been proved [Darrigol 2015, 172]. But a remark by Darrigol on § 22 of Einstein's 1916 review paper may open the door to an alternative view.

There, Einstein considers the "behavior" of rods and clocks in a static gravitational field. Einstein discusses a unit measuring rod, for which $ds^2 = -1$, laid in the radial direction, in which case $-1 = g_{11}dx_1^2$. According to Einstein, "the unit measuring-rod appears a little shortened in relation to the system of coordinates by the presence of the gravitational field" [Einstein 1916a, 197]. In this way:

Euclidean geometry does not hold even to a first approximation in the gravitational field, if we wish to take one and the same rod, independently of its place and orientation, as a realization of the same interval. [Einstein 1916a, 197]²¹

For the case of a unit clock "arranged to be at rest in a static gravitational field" [Einstein 1916a, 197], the time coordinate dx_4 is equal to $1 - \frac{(g_{44}-1)}{2}$. Accordingly:

The clock goes more slowly if set up in the neighbourhood of ponderable masses. From this it follows that the spectral lines of light reaching us from the surface of large stars must appear displaced towards the red end of the spectrum. [Einstein 1916a, 198]

21. In our view, this is a clear reference to a perverted geodesy. Compare Einstein's remarks related to the marble slab example or the rotating disk: (a) "The propositions of Euclidean geometry cannot hold exactly on the rotating disk, nor in general in a gravitational field, at least if we attribute the length 1 to the rod in all positions and in every orientation" [Einstein 1917, 334]; (b) "With reference to our little rods—defined as unit lengths—the marble slab is no longer a Euclidean continuum" [Einstein 1917, 338]. On this point, see the discussion on section 3. Note also that Einstein explicitly considers the rod to be shortened due to the presence of the gravitational field.

As we can see, the time coordinate is given a direct physical meaning as the time gone by on the clock under the influence of the gravitational field: this is incompatible with a concrete geodesy.

Regarding this determination by Einstein of “the influence exerted by the field of the mass M upon the metrical properties of space” [Einstein 1916a, 196], in particular with respect to the calculations regarding rods, Darrigol argues that:

This way of discussing the non-Euclidean properties of space is somewhat strange: On the one hand, it is based on the sound idea that true metric relations are defined by the metric element ds^2 ; on the other hand, it seems to retain a meaning for the concept of length with respect to a system of coordinates, as if the coordinates still had an independent physical existence. [Darrigol 2015, 172]

In terms of the “debate” between a concrete and a perverted geodesy in Einstein’s thought presented in this final section, we can read Darrigol’s remark in two ways:

- (A) Evidently, parts of Einstein’s calculations are based on sound mathematical properties of the Riemannian space-time, even if Einstein did not notice that this “imposes” a concrete geodesy (recall the final part of the previous section).
- (B) With the adoption of a metric field theory of gravity based on Riemann geometry and tensor calculus, Einstein adopts concrete geodesy—that is, he already does so in the context of the Entwurf theory. In this case, we might consider that Einstein’s standard position is *based on* concrete geodesy. However, Einstein *retains* perverted geodesy elements to present or even to obtain some important results.

On the evidence presented here, Darrigol’s remarks should be read as (A). As we saw in the final part of the previous section, the adoption of some basic features of a Riemannian space-time (like for a unit measuring rod having $ds^2 = -1$, for which, when laid in the radial direction, $1- = g_{11}dx_1^2$) does not necessarily imply that Einstein had any clear position based on concrete geodesy. On the contrary, as we have just seen, in §22 of his review paper we find references to a perverted geodesy. While we find (B) unconvincing, an even more nuanced view might be impossible to distinguish from ours in relation to the textual evidence taken into account here. Further studies may be needed for this, possibly from a perspective different from that adopted here.

Abbreviations

[CPAE] EINSTEIN, Albert (1987-), *The Collected Papers of Albert Einstein*, edited by John Stachel *et al.*, 14 vols, Princeton: Princeton University Press.

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