Philosophia Scientiæ

Travaux d'histoire et de philosophie des sciences

22-3 | 2018

Sur la philosophie scientifique et l'unité de la science

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Electronic version

URL: <http://journals.openedition.org/philosophiascientiae/1583>

DOI: 10.4000/philosophiascientiae.1583

ISSN: 1775-4283

Publisher

Éditions Kimé

Printed version

Date of publication: 25 October 2018

Number of pages: 119-134

ISBN: 978-2-84174-908-9

ISSN: 1281-2463

Electronic reference

Gabriele Lolli, "Federigo Enriques at the 1935 International Congress for Scientific Philosophy in Paris", *Philosophia Scientiæ* [Online], 22-3 | 2018, Online since 25 October 2020, connection on 31 March 2021.

URL: <http://journals.openedition.org/philosophiascientiae/1583> ; DOI: <https://doi.org/10.4000/philosophiascientiae.1583>

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Federigo Enriques at the 1935 International Congress for Scientific Philosophy in Paris

Gabriele Lolli

Scuola Normale Superiore di Pisa, Pisa (Italy)

Résumé : Au Congrès de philosophie scientifique de 1935 comme lors du lancement de l'Encyclopédie internationale de la science unifiée, Federigo Enriques était reconnu par les néo-positivistes comme un de leurs pères fondateurs, sans qu'il fût tout à fait d'accord. À Paris, Enriques représentait le groupe des philosophes des sciences italiens et son nom était lié au journal *Scientia*, ouvert aux contributions des positivistes logiques. Ces derniers, désireux de constituer un front commun pour lutter contre les philosophies idéalistes et métaphysiques alors dominantes, surestimaient sans doute la force du groupe italien, comme ils sous-estimaient les critiques d'Enriques sur l'usage de la logique formelle ou sur l'importance du langage. Dès les débuts de son implication en philosophie, il avait souligné la nécessité d'une fondation psychologique des concepts scientifiques, et en 1901, il avait donné un exemple de ce type d'analyse pour les postulats de la géométrie. Il n'appréciait guère les travaux logiques de Giuseppe Peano, affirmant que son formalisme manquait de bases psychologiques. Cependant, contre les mouvements anti-rationalistes de son temps, il défendait la valeur cognitive de la science et, dans ce climat d'appréciation mutuelle, les différences passaient au second plan.

Abstract: At the 1935 Congress for Scientific Philosophy, and at the launch of the International *Encyclopedia of Unified Science*, Federigo Enriques was recognized by the leaders of neo-positivism as one of their founding fathers, not to his complete agreement. In Paris, Enriques represented the Italian group of philosophers of science and his name was associated with the important journal *Scientia*, which was open to contributions by logical positivists. This latter movement, which wanted to create a front composed of the opponents of the prevailing idealistic and metaphysical philosophies, probably underestimated the criticism of Enriques with respect to formal logic and the importance of language. From the start of his involvement in philosophy in the first years of the twentieth century, Enriques had stressed the necessity of a psychological foundation for scientific concepts. He did not appreciate the

logical work of Giuseppe Peano, claiming that his formalism didn't have the right psychological basis. However, he defended the cognitive value of science against the anti-rationalist movements of the time, and in the climate of mutual appreciation differences were nuanced down.

1 Paris 1935, and thereabouts

In 1935, Federigo Enriques (1871-1946) spoke at the International Congress for Scientific Philosophy held in Paris, where the neo-positivist movement was launched; he gave not only a short talk [Enriques 1936b], but also an introductory address [Enriques 1936a], together with those of Charles W. Morris, Louis Rougier, Philipp Frank, Hans Reichenbach, Kazimierz Ajdukiewicz, and Bertrand Russell.

In Paris, as director of the journal *Scientia*, Enrique represented the only Italian group in the international panorama of scientific philosophy, and had been invited along with the Centre international de synthèse in France, the Vienna Circle, the Berlin Group, Anglophone and Scandinavian thinkers, so that all European nations in which there was some activity in the philosophy of science would be involved. Enriques was all in favor of the neo-positivist anti-metaphysical stance, but he had reservations regarding the insistence on the logical analysis of language. *Scientia* had, however, published papers by Moritz Schlick, Frank, Rudolf Carnap, Otto Neurath; Enriques himself had reviewed two booklets by Frank and Carnap, in [Enriques 1935].

In this review, Enriques acknowledged the agreement between the Vienna positivism and his own concerning the rejection of agnosticism and murky metaphysics, but he also stressed the distance between them as to the role of logical analysis. We will comment later on the origin and the nature of Enriques's position, but first let us note that at the Paris Congress he was explicit about his reservations. He introduced himself in the following way:

J'appartiens – moi – à la génération de ceux qui, élevés dans le milieu de la philosophie positive, ont vu, dans leur jeunesse même, se relever l'étendard de l'idéalisme métaphysique et engager une lutte violente contre l'esprit positif. Après trente années dominées par ces courants de la pensée, j'assiste aujourd'hui au renouveau de la philosophie scientifique, qui – à la vérité – n'a jamais cessé d'exister et d'être affirmée, pendant cette période, par des penseurs sortis du domaine des sciences particulières, mais qui, – depuis quelque temps – semble reprendre force, tendant à une domination nouvelle sur la culture.

C'est là un événement que je salue de tout mon cœur. [...]

Mais il ne s'agit pas de chanter victoire [...].

[L]a philosophie scientifique, en tant qu'elle aspire à établir une discipline supérieure de la pensée rationnelle, ne saurait se réduire à un système philosophique particulier, résolvant en un sens déterminé les oppositions traditionnelles des écoles. [Enriques 1936b, 23–24]

The short paper by Enriques [Enriques 1936b] was almost entirely devoted to offering cautionary advice against two forms of dogmatism. The first was the neglect of the investigation into the process of the acquisition of knowledge; he warned that “les préjugés empiristes ont induit en erreur notre penseur et historien éminent [Mach], dans le domaine même de l’histoire de la pensée” [Enriques 1936b, 25]. The second was logicism:

Les remarques qui précèdent, sous la forme d’une critique à une philosophie du passé, visent aussi les courants renouvelés de la pensée que le programme de notre Congrès semble confondre un peu avec la “philosophie scientifique” tout court; j’entends le “logicisme empirique”. Je prends beaucoup d’intérêt aux idées et aux critiques des philosophes éminents qui représentent cette école; mais je serais moins disposé à admettre que leur système constitue la seule philosophie vraiment scientifique.

[...] Je me défie davantage du logicisme. La raison qui construit la science, et qui se révèle par l’évolution historique de la pensée, ne saurait s’expliquer par une analyse purement logique [et deuxièmement, je me défie de l’ontologisme].

Des deux côtés je vois [donc] surgir devant nous le spectre d’une nouvelle scolastique. [Enriques 1936b, 26–27]

This notwithstanding, Enriques was appointed to the International Committee of the Congress for the Unity of Science, established in Paris, along with Carnap, Frank, Neurath, Reichenbach, Schlick, Morris, Niels Bohr, Percy Williams Bridgman, Abel Rey, Rougier, Russell, and Ferdinand Gonseth. More remarkably, two years later, Otto Neurath (1882–1945) invited him to contribute to the *International Encyclopedia of Unified Science* with an essay on the history of science. He had already been inserted in the genealogical tree of eminent ancestors of the neo-positivists—as reconstructed by Neurath in the manifesto of the Vienna Circle, written by Hans Hahn, Neurath, and Carnap [Hahn, Neurath *et al.* 1929], under the heading “Foundations, Goals and Methods of Empirical Science”—together with Hermann von Helmholtz, Bernhard Riemann, Ernst Mach, Henri Poincaré, Pierre Duhem, Ludwig Boltzmann, Albert Einstein, Giuseppe Peano (twice, under both “Logistic” and “Axiomatics”), Giovanni Vailati, Mario Pieri, David Hilbert, and others.

In a letter of February 4, 1937, Neurath asked Enriques to contribute with a paper of around seventy pages for the *International Encyclopedia of Unified*

Science;¹ upon a request by Enriques he agreed that it ought not to be a condensed history but a discussion of the importance of the history of science, and in particular of its logical importance. In the end the contribution would not be written; Enriques could not meet the deadline and in 1938 he suggested the name of Giorgio de Santillana (1902-1974) as a substitute, and in general as a collaborator for the *Encyclopedia*. In the same letter of February 4, 1937, Neurath invited Enriques to become a member of the Advisory Committee of the *Encyclopedia*, and Enriques accepted.

In a letter of June 18, 1937 Neurath recalled how Enriques's speech in Paris had touched them all, since, in it, he recalled his battle in support of empiricism during his youth and showed favor for the new empiricist movement, apart from some misgivings. After this captivating premise, he asked Enriques for two or three pages of introduction to the first issue of the *Encyclopedia*, dedicated to the unity of science, concurrently with John Dewey. He added that what they were planning brought all of them back to the recalled period of Enriques's youth, so that "this place certainly belongs to you". Enriques asked for more information and discussion on the matter. Neurath also invited Enriques to a restricted meeting that July to discuss the *Encyclopedia*, insisting that he give an opening address there.

Thus, the leading positivists, or those more actively involved in practical projects, began to court Enriques after the 1935 Congress; but behind the scenes they found it difficult to cope with his old-style rationalism, as can be retraced from the correspondence between the actors concerned.

Already during the preparation of the 1935 Congress, for example, Frank expressed to Neurath his discomfort at not knowing whether he was working for a congress of the Vienna Circle or for a general conference of philosophy of science [letter from Frank to Neurath, March 1935], the nuisance element coming especially from Louis de Broglie and Enriques. Later, Neurath admitted the difficulties encountered by Frank to reconcile all the collaborators of the *Encyclopedia*, among whom Enriques was the most prone to champ at the bit [letter from Neurath to Carnap, January 11, 1939]. It seems that the provisional prospectus was, in the end, written by Enriques, Rougier, and Frank, after discussions with Carnap, Jørgen Jørgensen and Neurath; but according to Neurath, in a letter to Morris [letter from Neurath to Morris, August 11, 1937], Enriques had been the most difficult to satisfy, as he wanted "to cover his back"; Neurath invites Morris to be generous in bringing into the picture references to tendencies such as pragmatism, instrumentalism,

1. The correspondence between Federigo Enriques and Otto Neurath is preserved in the Wiener Kreis Archive, corpus Neurath, at the Rijksarchief, Haarlem, North Holland, together with letters to and from Neurath and Philipp Frank, Charles W. Morris, Rudolf Carnap, among others. Enriques wrote in French, Neurath in German. It seems that Enriques corresponded only with Neurath, since his name does not appear in the *Guide to the Unity of Science Movement* of the University of Chicago.

commonsensism and rationalism, since “the more the picture is colourful, the more Enriques is pleased”.

Notwithstanding his fussiness, Enriques’s requests were always accommodated, although the scientific esteem was not entirely unreserved and flattering; in a letter to Frank [letter from Neurath to Frank, July 25, 1935] Neurath declared that as far as history was concerned Enriques certainly wasn’t the best choice.²

So why the courting? It is likely that, in terms of the new *Encyclopedia*, Neurath’s strategy aimed to balance the preponderant logical tendency with the historical one. Also, the fact that Enriques controlled the journal *Scientia* could have been a factor. Over the previous years, the journal had become a powerful instrument for the circulation and promotion of ideas.³ In the correspondence, the questions regarding Enriques’s participation in the activities of the *Encyclopedia* were mingled with those concerning Neurath’s collaboration with the journal.

There were, moreover, certain affinities between Neurath and Enriques, which probably made Neurath’s deference sincere; both acknowledged, for example, the importance of scientific education for social and civic emancipation; both felt the urge to organize cultural activities; neither viewed philosophy as a discipline, not even epistemology, only science mattered to them; both had read and enjoyed Wilhelm Wundt’s *Logik* [Wundt 1880-1883], where Neurath had found a clear statement of the unity of knowledge (these affinities are highlighted in [Simili 2000]).

By 1938, however, Enriques had to abandon his university position due to the racial laws that had been enforced, and since he remained in Italy he was precluded from engaging in any activity, except that which was undertaken

2. In a review of Enriques and Giorgio de Santillana (1932), George Sarton (1884-1956), after saying that Enriques “needs no introduction to our readers” describes him as belonging “like myself to the group of historians of science who have become historians only gradually and almost in spite of themselves under the increasing pressure of humanistic longings and philosophical perplexities” [Sarton 1935, 267–268].

3. In 1907, Enriques had been co-founder, along with Eugenio Rignano (1870-1930)—an engineer—of the *Rivista di scienza* (later—from 1910—known as *Scientia*), among whose contributors in the earlier years were Ernst Mach (in 1910), Henri Poincaré, Bertrand Russell, Albert Einstein, Sigmund Freud, and others. The journal followed the lead of the *Revue du mois* (founded the year before in Paris by a group of scholars led by Émile Borel) and of the *Annalen der Naturphilosophie* founded in 1901 by Wilhelm Ostwald in Leipzig. Because of disagreements regarding the space to be given to the problems of the war, Rignano became the only director of the journal in 1915, till his death in 1930, when, according to his expressed wishes, Enriques once again took on the role of director. *Scientia* had two versions, one international—published in Paris by Félix Alcan, in Leipzig by Wilhelm Engelmann, and in London by Williams and Norgate, where the articles were written in the language of their authors—and one Italian, where they were also translated into Italian. The Italian edition lasted in this form till 1988.

under a pseudonym. He was reinstated in his university chair in 1945. A final exchange with Neurath in the autumn of 1945 testifies to their mutual survival and their desire to go back to work, but neither had much time left.

2 The making of a philosopher

Enriques states in the preface to the book of 1906 [Enriques 1906] that gave him the status of a philosopher that his philosophical interests kept growing from 1890 to 1900; then, when he was sure of himself, he began to go public, probably referring to [Enriques 1901]. Since he graduated in 1891, this means that for the whole period in which he worked with Guido Castelnuovo (1865-1952) in Rome and Corrado Segre (1863-1924) in Turin on algebraic surfaces, becoming a worldwide-acclaimed geometer, he continued to reflect on philosophical matters. This is confirmed by the letters he exchanged with Castelnuovo: in 1896, for example, he confessed that he wasn't putting his heart and soul into mathematics, but rather into the philosophical problem of space [Enriques 1996, 260]. Nonetheless, he succeeded in producing the classification of algebraic surfaces with Castelnuovo during this period of time [Babbitt & Goodstein 2011]. He was appointed professor in Bologna in 1896, where he remained till 1922, when he moved to Rome.

He was by then considered an important philosopher, and not only in Italy. His 1906 book had been translated in the USA in 1914, in addition to previous French, German, Spanish, and Russian translations; in 1907, Enriques had co-founded the *Rivista di scienza* with Eugenio Rignano; in 1906, he had been among the founders of SFI, the Italian Philosophical Society, and had become its first president (till 1913), during which term he had organized the Fourth International Congress of Philosophy in Bologna in 1911.

Apart from his scientific prestige, however, as a philosopher he was respected abroad more as a charismatic figure, for his defense of rationality, than as a scholar. In Italy, he did not manage to attain any status at all outside of scientific circles since the acrimonious and disparaging critique of the idealist philosopher Benedetto Croce (1866-1952) had succeeded in isolating him (see references to [Croce 1981] in [Ciliberto 1982]).

The difficulty for philosophers of other languages and cultures to understand Enriques's theses and positions is made clear by Charles D. Broad's review of [Enriques 1914]; although the book is presented as containing interesting topics ("there are [...] some excellent remarks on the nature of definition" [Broad 1915, 96]), the review is interspersed with expressions of regret and complaint: "the style is very heavy [...]". The book is also disfigured by an immense number of notes of exclamation [...]. [I]t deals with so many difficult and important subjects that the argument is obscure through its condensation" [Broad 1915, 94], "Again I cannot see precisely what Prof. Enriques' special argument about the actual infinite is supposed to prove"

[Broad 1915, 95], “I am not perfectly sure that I understand this [...]” [Broad 1915, 97], and so on.

We will briefly describe Enriques’s philosophical thought through a summary of three of his major works, the psychological foundations of the postulates of geometry [Enriques 1901], the book *Problems of Science* [Enriques 1914], and that on the history of logic [Enriques 1929] (what follows is an abridged version of [Lolli 2012]).

2.1 Psychology and geometry

In 1901, as the result of years of reflections “of a mathematical, psychological, and philosophical order”, Enriques laid down his project, which was to give a psychological-genetic explication of the postulates of geometry. He viewed himself as belonging to a tradition comprising Johann Friedrich Herbart, Carl Friedrich Gauss, Nikolai Lobachevsky, the psychologists Alexander Bain and Joseph Delboëuf, Hippolyte Taine, and Hermann Lotze. According to Enriques, the physio-psychological experiments of Helmholtz and Wundt for showing the emergence of the representation of space by means of sensations and associations fell short of explaining the sense of necessity of the mathematical intuition of space; what was missing was a link between the postulates and the logical structure of thought. Enriques was a great admirer of Wilhelm Wundt (1832-1920), whose influence on Enriques is deep and transparent. For Wundt, logic was the study of “those laws of thinking active in scientific knowledge” [Wundt 1880-1883, vol. 1, 1, my translation]; symbolic logic was, in the *Logik*, a discipline apart, called “Logistik”.

Physio-psychological investigations had to be integrated by pure psychological research in order to show how concepts are formed from sensations, and to discover the necessary conditions of their genesis, dependent on the psychic structure.

The psychic structure is given by the laws of association, logical laws, and the sense of temporal order, i.e., distinguishing between “before” and “after”, prior to the sense of duration. Enriques was convinced that these capabilities were an expression of biological laws that were as yet unknown, and not the product of cultural inheritance [Enriques 1901, 81].

For Enriques “the three groups of representations linked to the basic concepts of the theory of the continuum (analysis situs), of differential geometry, and of projective geometry are immediately connected to three groups of sensations: respectively the tactile-muscular ones, those of touch, and those of vision”.⁴

For example, a line as viewed by the eyes either passes through the center of vision, and then it appears as a point, or not, and it is seen as a line; to relate

4. All translations from Italian are by the author, except for those from Enriques [Enriques 1906], which are taken from the English translation (undertaken in the U.S.), including references to page numbers.

these two representations it is necessary that the line be determined by any two of its points, not only as a postulate, but from a physiological determination. Enriques is convinced that the following is a sufficient explanation: looking from A at point B , the segment AB is defined as the set of points whose image falls in B ; the same set is viewed from outside as a segment and from B as a point. Hence, first of all, $AB = BA$. If C is on AB , then $AC = AB$ for the visual ray, and the same for another point D ; in conclusion $AB = CD$, that is, the line is determined by any two of its points.

The postulates of the line are the density axiom and the continuity axiom. To perceive the density already requires more than sensations: the concept of a line represents all possible successions of points; when two points are too near to appear distinct to the senses, the thought uses a correspondence with another line in which the two points are further off, so that in between a new point can be inserted.

Richard Dedekind's continuity postulate does not seem derivable from representations of the line as a succession of points; according to Enriques it could be based on two superimposed concepts, that of a corpuscle and that of the infinite divisibility of the line.

2.2 Problems of science

The general aim of the 1906 book was to propose a synthetic view of knowledge, opposing both the anti-rationalist tendency characterized by the *ignorabimus* of Emil Du Bois-Reymond (1818-1896) and positivist philosophy. By admission of its author, there is in the book a "relatively pragmatistic element", which was probably due to the influence of Giovanni Vailati (1863-1909), the philosopher whose untimely death in 1909 swept away Enriques's only Italian interlocutor; such an element, which Enriques labeled "critical positivism", has nothing to do, however, with William James's pragmatism and Henri Bergson's spiritualism, these being the thinkers who, in the years that followed, would originate a new anti-intellectualist movement, as Josiah Royce remarks in his preface to [Enriques 1914, x].

The book begins with "an analysis of what constitutes reality", and a defense or foundation of objective knowledge. Enriques tries to distinguish "the positive content of science on the one hand, and its subjective aspects on the other". Objective knowledge has different subjective versions if a prevision of the right answers is obtained by means of different images; the resort to some image is necessary, so pure objective knowledge does not exist. On the other hand, there is also an objective element in subjectivity, in that subjectivity can influence the previsions.

Knowledge is based on sensations, but not only or not precisely on sensations, rather on the correspondence of sensations to voluntary acts.

There are fixed groupings, independent of us, amongst our actual or supposed volitions on the one hand, and the sensations

produced by them on the other hand [...]. These groupings correspond to what we call “the real”. The real gets defined, in this way, as an invariant in the correspondence between volitions and sensations. [Enriques 1914, 65]

In an analogous way, a scientific fact is a relation of “order of succession” between facts conceived as invariant. For example, “[a scientific fact] teaches us that we can experience the sensations that testify to the heating up of the plate of metal, whenever we perform the acts that produce the blow of the hammer against it” [Enriques 1914, 67]. A scientific fact is actually an abstraction, realized by the elimination of data to obtain a “type of a series of possible facts”, a type that Enriques calls a concept.

A theory is the subjective aspect of the knowledge of a scientific fact, and precisely “a concept or system of concepts [...] from which, by deduction, we can subordinate [other] supposed facts”.

There arise two classes of problems:

- (1) the problems connected with the logical transformation of concepts, regarded both as a psychological development and as an instrument of knowledge; and (2) the problems that refer to the significance and to the acquisition of the more general concepts of space, time, force, motion, and so on. [Enriques 1914, 67]

The latter are treated in the second part of the book.

Logic is discussed at length in the third chapter, covering more than one hundred pages [Enriques 1914, 153–259]; it is the real focus of Enriques’s reflection, as confirmed by his second important book of 1922 [Enriques 1922].

As was generally the case with him, Enriques contextualizes his own considerations in a historical perspective. The thinkers of the Middle Ages were mistaken in believing that they could attain the truth by studying formal rules alone. The study of reasoning has been enlarged in modern times to include the treatment of empirical data, providing an origin to what is now called inductive logic. However, Enriques is not interested in the latter.

Formal logic is only an instrument, it does not change the data of knowledge, but it is essential because, as William Stanley Jevons remarked, it transforms hypotheses that are inaccessible to experience into hypotheses that are equivalent but that can be verified and possibly refuted by experience. Formal logic is important, the question is whether it is possible.

With regard to the endeavor of trying to establish a foundation for a formal logic, Enriques discusses two methods. The first one is that pursued so far by scholars, and it consists in isolating verbal schemes corresponding to ideal forms and in eliminating from them all ambiguities. For example, the word “some” in common talk frequently means “some but not all”, but this meaning has been eliminated from its scientific use. This path leads to symbolic logic, perfected in a tradition in which Enriques includes Gottfried

Leibniz, Johann Heinrich Lambert, George Boole, Ernst Schröder, Giuseppe Peano, and Gottlob Frege.

The second method consists in a study of the actual functioning of thought; psychological logic is an empirical science, falling under psychology. This is why Enriques didn't mention logic when he described the range of his studies in 1901, as we have seen above. In 1901, Vailati told Giovanni Vacca (1872-1953), a student of Peano, that Enriques was studying logic, but he didn't understand what kind of logic it was, certainly not mathematical logic ([Vailati 1971, 188]; by "mathematical logic" Vailati meant Peano's logic).

Definitions and deductions are to Enriques psychological operations; their combination provides the logical process with an origin. Logical processes are those mental processes in which some conditions of consistency are voluntarily satisfied.

In order to give definitions, one must suppose that some objects are given in thought, capable of entering into some associations that satisfy a sort of invariance. Psychological operations are, in fact, associations and dissociations, which allow for the building of new objects. From the association of objects, either simultaneously or in succession, we acquire the operations of conceiving classes and series; comparing classes gives rise to the concept of correspondence. Dissociations, on the contrary, give the inversion of series, the disjunction, the intersection and the abstraction, by which an element of an abstract class is substitutable for any other element. Among propositional laws, for example, the associative and commutative ones come from the psychological operation of making a heap.

The invariance of objects with respect to operations is expressed by the principles of identity, non-contradiction, and excluded middle; this latter principle means that for two objects a judgement regarding identity or diversity is always possible. It is based on the fundamental intuition of "before" and "after". Such intuition is not representable by formulae; one should not write $a = a$.

Enriques admits that the working of the logical process can be represented by symbolic logic as a system of actual relations, thus conferring on the objects of thought a reality independent of time. One would thus expect an eventual convergence of the two methods of founding formal logic, but this is not so, as we shall see in the criticism of Peano's ideography.

The logical relations among objects built via thought are a-priori compatible. As to arithmetic, consistency derives from the fact that acts of thought can be repeated in an unlimited way. Enriques, however, calls this statement a psychological postulate. Thus, arithmetic forms part of pure logic.

When one reasons, one assumes as evident that the reality of the premises entails that of the conclusions; the transformation of concepts determined by subjective laws reflects or becomes a transformation of real relations. In order for this to be possible there must be a relation between the invariants of experience and those of thought. However, a word of caution should be added:

the applicability of logic assumes as a postulate that under the conditions of invariance expressed by the logical principles, the systems of things satisfy only approximately the properties expressed by the axioms.

In the light of such a postulate, it is curious that Enriques had such a dislike for Russell; he disapproved of his idea that logical relations express the most general relations holding among entities of every possible world; thus, they wouldn't relate to the analysis of thought but would be truths in a metaphysical universe. Hence, the Russellian curse of "the totally irrelevant notion of mind" [Russell 1903, chap. 1, § 3]. To characterize his own position in opposition to Russell's, Enriques resorted to Boole, for whom logic was the set of laws guiding mental processes, which can only have a representation in the static image of a formalism.

It is not clear whether the approximation in the correspondence between the invariants of the thought and the invariants of reality can be related to some fundamental character of definitions. According to Enriques it is impossible to establish the real meaning of a concept through a definition; definitions, at least implicit ones, do not give an exact determination but throw only a partial light on the real sense of concepts. They have to be completed with a concrete interpretation through suitable rules of correspondence. This idea is related to the axiomatic method, which is discussed at greater length by Enriques in [Enriques 1922].

At the end of the chapter on logic Enriques summarizes a few neurological findings that were known about brain functioning at this time, expressing the hope that his own approach could be confirmed in the future as a result of advances in this field.

2.3 The history of logic

In the 1922 book the analysis of logic is enlarged, and it becomes clear what Enriques meant in the *Problems* by saying that the possibility of formal logic follows from the development of mathematics. The subtitle of the English translation of the book, published in 1929, is *The Principles and Structure of Science in the Conception of Mathematical Thinkers*. The structure, or structuring (in Italian *ordinamento*) of science is now an obsolete concept, but central to Enriques's vision.⁵ The problems of the structuring of theories concern the definitions, the axioms, the place they have in the body of the theory, the criteria of their choice, and how to judge their acceptability. These are the problems that should be discussed in the study of logic. Through his analysis, Enriques qualifies as one of the most sensible and clear-minded interpreters of the modern formal axiomatic method.

5. It has been rendered as "structure" in the American translation [Enriques 1929]. There have been also French and German translations.

The structure of the sciences for the Greeks displayed a naive realism, the necessary character of the principles, no theory of definitions, and a concept of deduction based on the meaning of the terms.

Enriques underlines some of Leibniz's contributions to the reform of logic at the beginning of the modern age: the idea that a concept is not tied to the real but to the realm of the possible; the principle of sufficient reason, from which the existent is singled out among the many possible.

Notwithstanding the break with the ancient tradition, the development of logic up to the nineteenth century had not changed the traditional concept of the structuring of demonstrative sciences. This task had to be accomplished by mathematicians: several intellectual movements with different origins concurred regarding the same reform. Enriques mentions projective and non-Euclidean geometries, Riemann, Eugenio Beltrami's models, the formal algebra and logic in Great Britain, the arithmetization of analysis, the new trend in physics towards the building of models. But only through the recent critique of the principles of geometry "the mathematical thinkers acquire a full conscience of a revolution harbored for centuries" [Enriques 1922, 132]. The final phase for Enriques begins with Joseph Diez Gergonne (1771-1859), his notion of implicit definition and the duality principle as a principle of substitutivity of concepts. It comes into full bloom with the geometric work of Julius Plücker (1801-1868) and Karl von Staudt (1798-1867): with Plücker's coordinates, a unified treatment of correlated entities leads to one form of intuition translating into other forms.

Enriques believed that multiple interpretations were an added benefit to the axiomatic method ever since he began to reflect on the topic:

The importance that we attribute to Abstract Geometry is not [...] in opposition to the importance attributed to intuition: rather, it lies in the fact that Abstract Geometry can be interpreted in infinite ways as a concrete (intuitive) Geometry by fixing the nature of its elements: thus, Geometry can be assisted in its development from infinite different forms of intuition. [Enriques 1894-1895, 9-10]

He then glorifies the method in light of this possibility:

Nothing is more fecund than the multiplication of our intuitive powers as enhanced by this method: it is almost as though to our bodily eyes—with which we examine a figure under a certain perspective—a thousand spiritual eyes were to be added, allowing us to contemplate several different transfigurations, while the unity of the object is resplendent under the enriched reason. [Enriques 1894-1895, 9-10]

The notion of abstract theory is thus presented as such: some primitive concepts— A, B, C —are given; a postulate states a certain relation— $f(A, B, C)$ —among them; when we ask whether the relation is true or false

for some new interpretation, the translation makes no sense if the relation f depends on the intuitive meanings of A, B, C . Hence the formal nature of mathematics. When two systems— R and S —are both possible interpretations of the same abstract theory, if, to discover some consequences, we look at the objects given with R , we must be careful that they do not depend on particular intuitions that are falsified in S .

Enriques mentions Moritz Pasch (1843-1930) as one of the thinkers who have contributed to the definition of the new structure of geometric theory, along with David Hilbert (1862-1943) and Giuseppe Peano (1858-1932). In [Enriques 1921], however, Enriques uses strong words of irony and contempt against Peano's ideography.

Enriques's criticism focuses on the following technical idea, that there is a fatal shortcoming in Peano's logic in the impossibility of explaining the following paradox: Peter was an Apostle, the Apostles were twelve, so Peter was twelve. Peano's way out consists in a distinction between membership and inclusion, but this move is contrary to the use of "is" in common speech. Enriques is strongly opposed to it; his own solution to the paradox is that the middle term "Apostle" has a double status, appearing once as a class and once as an abstract concept.

In the lemma "Mathematical logic" written for the *Enciclopedia italiana* by Beppo Levi [Levi 1934], an unsigned appendix bearing the title "Meaning of logic", makes a comparison between Peano and Enriques; it can be confidently surmised that the appendix was written by Enriques himself, who was in charge of the mathematical entries of the *Encyclopedia*:

Giuseppe Peano and his followers see in the logical symbolism an ideography well suited to the exposition of the 'deductive and mathematical sciences'; from the positive development of these sciences they argue a sort of experimental revelation of the logical schemes of reasoning, which logic is meant to analyze. No further question on the meaning of reasoning, and hence on that of logic, is considered by this school. To Enriques, logic is the study of the operations of the exact thought and their laws, without reference to anything outside the mind. [Levi 1934, 401]

There follows a short presentation of Enriques's ideas on the logical process, and finally Enriques's critique of Peano's distinction between membership and inclusion is repeated:

a classroom conceived as a union of individuals is something different from the abstract concept of the classroom: the union of students, $B, C \dots$ gives the school; from this, by abstraction, one gets the student (of that school) [...]. Ordinary language makes here a distinction that is impossible to translate in Peano's symbolism, and as a consequence he has to distinguish instead two meanings in the copula of ordinary language. [Levi 1934, 401]

3 Conclusions

Enriques's philosophical ideas were formed at the end of the nineteenth century and didn't evolve. He was a prolific writer, but he did not confront head on the main new trends in philosophy and in logic.

One shouldn't forget that most of Enriques's activity was devoted to mathematics, and, after his juvenile years, especially to research and contributions in mathematical education: in Rome he held a chair in a new discipline conceived for teachers' training; in 1921, he took on the dedicated journal *Periodico di matematiche*, renewed and directed it till 1938, and wrote a lot of textbooks (literally dozens) both about geometry and elementary algebra and about higher mathematics. He was also actively involved in the discussions on the reform of universities (as documented by [Giacardi 2012]).

I would like to share one fair, final assessment that was proposed by the historian of mathematics Jeremy Gray :

It is harder for us today to accommodate his writing as a philosopher or populariser. He held a subtle position, according to which knowledge is inseparable from the means of knowing, logic from psychology. This has long been unfashionable in the sciences. It may be that cognitive psychology will reopen the avenues Enriques explored; there are signs that it has reached at least the philosophy of mathematics. [Gray 1996, 54]

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