Philosophia Scientiæ

Travaux d'histoire et de philosophie des sciences

25-3 | 2021

L'analyse dans les mathématiques grecques

Aristotle and Greek Geometrical Analysis

Enrico Berti

Translator: Gianluca Longa

Electronic version

URL: <https://journals.openedition.org/philosophiascientiae/3185>

DOI: 10.4000/philosophiascientiae.3185

ISSN: 1775-4283

Publisher

Éditions Kimé

Printed version

Date of publication: 25 October 2021

Number of pages: 9-21

ISBN: 978-2-38072-041-9

ISSN: 1281-2463

Electronic reference

Enrico Berti, "Aristotle and Greek Geometrical Analysis", *Philosophia Scientiæ* [Online], 25-3 | 2021,

Online since 25 October 2021, connection on 06 November 2021. URL: <http://journals.openedition.org/philosophiascientiae/3185> ; DOI: <https://doi.org/10.4000/philosophiascientiae.3185>

Tous droits réservés

Aristotle and Greek Geometrical Analysis

Enrico Berti

University of Padua

Translated by Gianluca Longa¹

Résumé: Cet article vise à montrer dans quelle mesure l'examen de certains passages de l'œuvre d'Aristote peut contribuer à la résolution d'importants problèmes liés à l'interprétation de l'analyse géométrique ancienne. Nous nous concentrons notamment sur le célèbre passage des *Seconds Analytiques* dans lequel Aristote se réfère de manière apparemment cryptique à l'analyse pratiquée par les géomètres (*Seconds Analytiques*, livre I, chapitre 12) et nous montrons l'importance fondamentale de ce passage pour une compréhension correcte de l'analyse géométrique ancienne.

Abstract: This paper aims to show that an examination of some passages in Aristotle's work can contribute to the resolution of crucial problems related to the interpretation of ancient geometrical analysis. In this context, we will focus in particular on the famous passage of the *Posterior Analytics* in which Aristotle cryptically refers to the analysis practised by the geometers (*Posterior Analytics*, book I, chapter 12) and we will show the fundamental importance of this passage for a correct understanding of ancient geometrical analysis.

In an appendix of the Book XIII of Euclid's *Elements*, before some alternative demonstrations of propositions 1–5 of the same book [EE IV,

1. This article was originally published in Italian as “Aristotele e l'analisi matematica greca”, in A. Repola Boatto (ed.), *Filosofia, logica, matematica dal periodo classico al nostro secolo*, Quaderni di “Innovazione scuola”, 1994, pp. 41–53 (reprinted in E. Berti, *Nuovi studi aristotelici*, vol. 1 : Epistemologia, logica e dialettica, Morcelliana, Brescia, pp. 113–125). We would like to thank both the author and the publisher for the permission to publish the translation of the article in this volume.

364.18–376.22], we find the oldest surviving description of the mathematical procedures of analysis and synthesis:

ἀνάλυσις μὲν οὖν ἐστὶ λήψις τοῦ ζητουμένου ὡς ὁμολογουμένου διὰ τῶν ἀκολούθων ἐπὶ τι ἀληθὲς ὁμολογούμενον. σύνθεσις δὲ λήψις τοῦ ὁμολογουμένου διὰ τῶν ἀκολούθων ἐπὶ τὴν τοῦ ζητουμένου κατάληξιν ἥτοι κατάληψιν.

Analysis is the assumption of what is sought, as if admitted, through the consequences, up to something admitted as true. And synthesis is the assumption of what is admitted, through the consequences, up to the ending up or apprehension of what is sought.¹

This description has raised at least four problems, the first of which involves its text. The version reported here, being the most intelligible, is in fact transcribed only in some medieval manuscripts—the *Bodleianus* scr. 888 (siglum **B**), the *Vindobonensis* XXXI 13 (siglum **V**), the *Bononiensis* (siglum **b**) and the *Parisinius* Gr. 2344 (siglum **q**). Though the English translator of the *Elements*, Th. L. Heath, considered it as authentic [Heath 1908, III, 442], there exists a variant, in a 9th century manuscript—the *Vaticanus* Gr. 190 (siglum **P**)—in which the definition of the analysis is the same, whereas the one of the synthesis is different:

σύνθεσις δὲ λήψις τοῦ ὁμολογουμένου διὰ τῶν ἀκολούθων ἐπὶ τι ἀληθὲς ὁμολογούμενον.

And synthesis is the assumption of what is admitted, through the consequences, *up to something admitted as true*.

This variant was preferred by the first modern editor of the *Elements*, J. L. Heiberg [*EE* IV, 366.2]. Therein, the final part of the definition of the synthesis is identical with the final part of that of the analysis, which is seemingly inexplicable. As known, we dispose of two versions of Euclid's *Elements*, one edited by Theon of Alexandria in the 4th century AD, and an older one. **P** reports the latter, while **B**, **V**, **b**, and **q** reproduce the Theonine version [Heiberg 1903, 46]. The variant in **P**, which indeed raises difficulties, is therefore the oldest, while the text presented in **B** and **V** seems to be more like a revision done to make sense of the definition of the synthesis; it reflects, probably, a rewriting of Theon himself. According to Heiberg, however, the whole appendix in question—the two definitions and the alternative proofs of the first five propositions of Book XIII—represents an interpolation, being completely extraneous to the way of exposition of the *Elements*.² According to him, the appendix might have been initially in the margin, in the

1. Classical quotations are directly translated from Greek into English. [Translator's note]

2. "Nam hoc totum interpolatum esse, certissimum est, quippe quod toto genere a ratione institutoque Elementorum abhorreat" [*EE* V, LXXXIV].

form of a scholium, as allegedly confirmed by the fact that, even in **V**, only its first part is placed in the main text, while the remaining is transcribed in the margin [*EE* V, LXXXIV].

The second problem concerns the origins of the two definitions. Bretschneider [1870, 167–169], by relying on Proclus’s passage stating that Eudoxus “multiplied the number of propositions concerning the ‘section’ [περὶ τὴν τομῇν] which had their origin in Plato, employing the method of analysis for their solution” [*EE*, 67.6–8], argued that they date back to Eudoxus of Cnidus, a contemporary and collaborator of Plato, author of many of the doctrines contained in the *Elements*. At first, Heiberg endorsed this thesis, indicating as possible authors of the definitions, either Eudoxus or Theaetetus, the mathematician contemporary of Plato after whom the homonymous dialogue is named. He then modified this view, after the publication of the commentary by al-Nairīzī (who lived around AD 900) on the first ten books of the *Elements* in the Medieval Latin translation by Gerard of Cremona. In this text, we find several quotations from the lost commentary of Heron of Alexandria (1st century AD) on the *Elements*, one of which reports two definitions of analysis and synthesis:

Dissolutio autem est, <ut>, qualibet questione proposita, primo ponamus illud in ordine rei quesite, que est inventa, deinde reducemus <ad illam>, cuius probatio iam precessit. Tunc ergo manifestum dicimus, quod iam inventa est res quesita secundum dissolutionem. Compositio vero est, ut incipiamus a re nota, deinde componemus, donec res quesita inveniantur. Ergo tunc res quesita iam erit manifesta secundum compositionem.

Now analysis is when, to any question proposed, we first set it in the order of the thing sought, which has been found, then we reduce it to something whose proof has already preceded. Then it is manifest that the thing sought was found according to the analysis.

And synthesis is, when we begin from a thing known, then we compose, until the thing sought is found. Thus the thing sought is manifest by synthesis.³ [al-Nairīzī 1899, 89.6–20]

Heiberg deemed this text very similar to that of the *Elements* [Heiberg 1903, 58–59] and concluded, followed then by several other scholars,⁴ that both definitions were to be ascribed to Heron. As shown in [Berti 1984, 93–127], the similarities detected by Heiberg are not so striking to induce to consider

3. This text should be compared with the scholium n.82 of book XIII of Euclid’s *Elements* [*EE* V, 675.12–18]: “τί ἐστι τὸ κατὰ ἀνάλυσιν; ὅταν προβλήματος δοθέντος λάβῃ τις τὸ ὡς εὑρημένον καὶ ἀναλύσῃ ἐπὶ τι γνώριμον τῶν ζητούμενων προαποδεδειγμένων, καὶ ἤδη, λελύσθαι λέγεται τότε εὕρη πρόβλημα ἀνάλυσιν: Τί ἐστι τὸ κατὰ σύνθεσιν; ὅταν τις ἀπὸ τῶν γνωρίμων ἀρξάμενος καὶ συνθεὶς εὑρηται τὸ ζητούμενον.” [T. N.]

4. [Heath 1908, III, 442], [Mugler 1948, 297], [Hintikka & Remes 1974, 30, n. 4].

Heron as the author of the two definitions. After all, the teaching of analysis in Antiquity was commonly attributed to Plato, as witnessed by the *Index of Academic Philosophers* [Mekler 1902, 17], Diogenes Laertius [VP III, 24], and Proclus [*iEE*, 211.21–22].⁵ However, the problem of establishing whether the definitions of analysis and synthesis are pre-Euclidean or post-Euclidean remains unsolved.

The third problem, related to that of the origin of the two definitions, is about the “direction” of analysis. It was first raised by F.M. Cornford [Cornford 1932, 37–52 and 173–190], who claimed that analysis and synthesis should be identified with the two procedures described by Plato in the Books VI and VII of the *Republic*, one that goes up from the “hypotheses” to the “unhypothetical principle”, called “upward movement”, and the one that goes down from the hypothetical principle, i.e., from the idea of good, to the other ideas, called “downward movement”. Against this argument, R. Robinson observed that the “upward movement” described in the *Republic* is presented by Plato as a dialectical procedure, not as a mathematical one, and that analysis, in all the extant definitions, is always described as a procedure passing through “consequences” (ἀκόλουθα in Greek), that is, as a “downward movement” [Robinson 1936, 464–473]. According to Robinson, the difference between analysis and synthesis, is not in the direction, which is always the same, i.e., “downward”, but in the fact that analysis starts from a sought proposition, taken as a hypothesis, and deduces a *known* proposition as a consequence, whereas synthesis starts from known propositions and deduces a *sought* proposition as a consequence. The problem is complicated by the fact that, in the definitions of analysis and synthesis proposed by Pappus of Alexandria [*PAC* II, 634–636]—who greatly influenced some of the most important early-modern philosophers like, for instance, Galilei and Descartes—there are some clues in favour of the “downward” interpretation and others in favour of the “upward” interpretation. This has led some scholars to suppose that Pappus’ description is contradictory [Gulley 1958, 1–14], and others to condemn as spurious the passage in favour of the “upward” interpretation [Mahoney 1968, 319–348]. Others again⁶ have denied that the Greek term ἀκόλουθα means “consequences” and stated that it’d rather mean “concomitants”, so it can denote both an “upward” and a “downward” reasoning.

The last problem, raised by Hintikka & Remes [1974, 31–40], concerns the question of whether analysis applies to propositions or figures. In geometry, the “constructive” meaning of analysis—that concerning figures and, more specifically, the finding of auxiliary constructions—is in fact the most important. Nevertheless, the expression “something admitted as true”

5. Heath [1921, 290–291], moreover, like others before and after him, like [Tannery 1915, 162–169], [Milhaud 1934, 175–176], and [Cambiano 1967, 115–149], endorsed the thesis that geometrical analysis was already practised, before Plato, by the Pythagoreans and by Hippocrates of Chios (5th century BC).

6. [Hintikka & Remes 1974, 7–21], [Szabó 1974, 118–130].

[τι ἀληθὲς ὁμολογούμενον], used in the Euclidean definition to indicate the point of completion of the analysis, seems to refer more to a proposition than to a figure.

1 Aristotle's clarifications on analysis

A decisive contribution to the solution of the four problems outlined above might be gained from a passage devoted to geometrical analysis contained in Aristotle's *Posterior Analytics* [78a6–12]:

εἰ δ' ἦν ἀδύνατον ἐκ ψεύδους ἀληθὲς δεῖξαι, ῥηξέδιον ἂν ἦν τὸ ἀναλύειν· ἀντέστρεφε γὰρ ἂν ἐξ ἀνάγκης. ἔστω γὰρ τὸ *A* ὄν· τούτου δ' ὄντος ταδὶ ἔστιν, ἃ οἶδα ὅτι ἔστιν, οἷον τὸ *B*. ἐκ τούτων ἄρα δείξω ὅτι ἔστιν ἐκεῖνο. ἀντιστρέφει δὲ μᾶλλον τὰ ἐν τοῖς μαθήμασιν, ὅτι οὐδὲν συμβεβηκὸς λαμβάνουσιν (ἀλλὰ καὶ τούτῳ διαφέρουσι τῶν ἐν τοῖς διαλόγοις) ἀλλ' ὀρισμούς.

If it were impossible to prove truth from falsehood, then analysis would be easy; for [propositions] would necessarily convert. Let *A* be true: if this is true, then these things, for instance *B*, are true—things which, in fact, I know to be true. From the latter, then, I will prove that the former is true. Now, mathematical [propositions] convert more often, because they do not assume accidents—even in this they differ from those in the discussions—but definitions.

The sentence opening this passage is clearly an unreal conditional: according to Aristotle, it is possible to demonstrate the truth from the false, just as it is possible to prove the truth from the truth and the false from the false. The only impossibility is that of proving the false from the truth [*An. Pr.* I, 2-4]. Hence, analysis is generally “difficult”. But let us follow Aristotle's reasoning and sketch what would happen if—an unreal hypothesis—it were impossible to prove the truth from the false, that is, if the truth could only be proved from the truth. In this case, analysis would be easy because “[propositions] would necessarily convert” [ἀντέστρεφε γὰρ ἂν ἐξ ἀνάγκης]. Indeed, the convertibility of a proposition consists in the possibility of exchanging the subject and the predicate, without affecting the truth-value of the proposition itself. When this happens, Aristotle observes that it is not only possible, starting from a proposition *A* to prove a proposition *B*, but also to prove *A* starting from *B*. In other words, the convertibility ensures the reversibility of the passage from premises to the conclusion and from the conclusion to premises, as it would happen if it were impossible to prove the truth from the false, i.e., if the truth could only be proved from the truth.

To illustrate this point, we can imagine the following syllogism:

“*A* is *B*”

“*B* is *C*”

∴ “*A* is *C*”

If the proposition “*B* is *C*” is true and convertible, then we can plainly say that “*C* is *B*” is also a true proposition. This allows us to set a syllogism in the opposite direction, by taking “*A* is *C*” as a premise and obtaining “*A* is *B*” as a conclusion:

“*A* is *C*”

“*C* is *B*”

∴ “*A* is *B*”

In such a case, we are not only able to prove the conclusion from the premises, but we can also prove one of the premises from the conclusion and the other premise, once converted. In this way, we obtain a situation in which, according to Aristotle, analysis is easy.

If we apply this observation to the Euclidean definitions of analysis and synthesis, we can readily see that the propositions denoted by *A* and *B* in Aristotle’s text correspond respectively to the expressions “that which is sought” and “something admitted as true” in the Euclidean definition of analysis. Thus, the sense of Aristotle’s remark is the following: if it were impossible to prove the true from the false, the propositions could necessarily be converted. This means that, once proved *B* from *A* by analysis, one’s can straightforwardly prove *A* from *B* by the synthesis. In such a situation, analysis is easy insofar as it permits to achieve those true propositions that, by synthesis, prove what is being sought. Aristotle remarks then that “propositions in mathematics convert more often because they do not assume accidents [...] but definitions” [ἀντιστρέφει δὲ μᾶλλον τὰ ἐν τοῖς μαθήμασιν, ὅτι οὐδὲν συμβεβηκὸς λαμβάνουσιν [...] ἀλλ’ ὁρισμούς]. With these words, he indicates in which cases propositions are convertible and in which they are not. Definitions are always convertible because there is a perfect identity between the subject and the predicate in them, which can therefore be exchanged without any alteration of the truth-value of the proposition obtained by conversion. On the other hand, propositions consisting in the assumption of an accident are not convertible because in them, there is no identity between the subject and the predicate and their conjunction is entirely contingent. Propositions used in mathematics are usually definitions, in which there is an identity between subject and predicate, or equations, and in fact mathematicians use the algebraic sign of equality (=) instead of the copula “is”, or propositions which indicate a property of the subject, a predicate which necessarily belongs to it, and not an accident. Therefore, mathematical propositions are usually convertible. But this is not always the case. Even

in mathematics, there may be propositions that cannot be converted, and for this reason analysis may be difficult [*Soph. El.* 175a 25–27]:

συμβαίνει δέ ποτε καθάπερ ἐν τοῖς διαγράμμασιν· καὶ γὰρ ἐκεῖ
ἀναλύσαντες ἐνίοτε συνθεῖναι πάλιν ἀδυνατοῦμεν·

At times, it happens in geometrical propositions: there too, after
doing the analysis, we are sometimes unable to synthesise back.

Since, in mathematics, propositions are not always convertible, analysis may fail in its purpose. It is nevertheless clear that propositions are convertible “especially” in mathematics, and that analysis is therefore especially easy in this discipline.

In the passage from the *Posterior Analytics* quoted above, Aristotle makes a further observation: the fact of assuming definitions and not accidents is one of the differences between mathematics and “discussions” [διάλογοι]. With the latter expression, Aristotle indicates the whole field of the non-mathematical discussions, governed by the technique he calls “dialectics”, which is sometimes used by him also in particular non-mathematical disciplines, such as physics and ethics. Analysis can also be used in these fields, but it is not equally easy to handle, since it is not as reliable as it is in mathematics. Aristotle provides two examples of the use of analysis in non-mathematical fields. The first concerns physics and, in particular, the notion of “hypothetical necessity” existing in the natural world, once this is conceived as a finalistic order. In this realm, as Aristotle states there is only a hypothetical necessity: if an end happens, the means must also happen, but on the contrary, if a means happens, the end does not necessarily happen. For example, if the house is the end and the bricks are the means, we can say that, if there is a house, bricks must also be there. On the other hand, we cannot say that if there are bricks, there *must* be a house [*Phys.* II.9, 200a 5–30]. Here we are dealing with a relation that is necessary only in one direction that which goes from the end to a means, exactly as it happens in analysis, when it takes place between non-convertible propositions. Therefore, we may say that once the end is known, the search for the means is a procedure of analysis. This is not necessarily followed by a synthesis, i.e., by a demonstration of the end based on the means. Aristotle does not explicitly mention analysis here, but the process he describes, as noted by various scholars⁷, corresponds perfectly to it.

The second example deals again with the relationship between the end and the means in the domain of ethics, and more precisely in the analysis of human action. It is exposed in the comparison made between “deliberation” [βούλευσις] and geometrical analysis in the *Nicomachean Ethics* [III.3, 1112a 18–28]:

ὁ γὰρ βουλευόμενος ἔοικε ζητεῖν καὶ ἀναλύειν τὸν εἰρημένον τρόπον
ὥσπερ διάγραμμα.

7. [Mugler 1948, 331–351], [Gulley 1958, 8], [Heath 1949, 100–101].

[The person] who deliberates seems to inquire and analyse in the way described, as though with a geometrical proposition.

For Aristotle, deliberation is not about ends, but about the means necessary to achieve them: in deliberation the end is presupposed, just as “what is sought” is presupposed in geometrical analysis. Means make possible to realise the end, just as in analysis the “consequences” are what enable to demonstrate what is sought. The mean found at the end is the first cause as in analysis the true principle reached at the end is the premise from which it is possible to prove the consequences up to what is sought. Here the procedure of analysis is explicitly mentioned, always in connection with a relation necessary only in one direction.

In physics and ethics, therefore, it is possible to use analysis. Nevertheless, here analysis is not always reliable. Both deal in fact with connections that are not necessarily in both directions, but, in the case of the natural world, apply only “for the most part” and, in the case of ethics, concern “things that can also be otherwise”, i.e., that depend on human freedom. In mathematics, on the other hand, the consequences of a premise “always” occur when connections are necessarily in both directions. Therefore, in this context, analysis is usually reliable.

2 Aristotle’s observations and the Euclidean definitions

In the aforementioned passages, as we have seen, Aristotle reveals a keen awareness of the *modus operandi* and import of geometrical analysis, partially disproving the common saying, dating back to Galileo [Galileo Galilei 1842, I, 302], whereby Aristotle was *parum in geometria [...] versatus* [little versed in geometry]. As T. L. Heath has argued⁸ although Aristotle did not make any personal contribution to the development of mathematics, he had a broad knowledge of the mathematics of his time, viz. the mathematics practised in Plato’s Academy and later systematised in Euclid’s *Elements*. It might be added that Aristotle regarded mathematics as a discipline already brought to excellence within the Academy—where he himself lived for twenty years—to

8. “Aristotle was evidently not a mathematician by profession, but he was abreast of the knowledge of his day [...] He was aware of the important discoveries of Eudoxus which affected profoundly the exposition of the *Elements* by Euclid. One allusion clearly shows that Aristotle knew of Eudoxus’ great Theory of Proportion which was expanded by Euclid in his Book V, and recognized the importance of it. Another passage recalls the fundamental assumption on which Eudoxus based his “method of exhaustion” for measuring areas and volumes; and, of course, Aristotle was familiar with the system of concentric spheres by which Eudoxus and Callippus accounted theoretically for the independent motions of the sun, moon, and planets” [Heath 1949, 1–2].

the extent of taking it as a model, especially in the *Posterior Analytics*. But the passages examined above also represent a major contribution to the solution of the issues raised by the Euclidean definitions of analysis and synthesis.

They provide an immediate answer to the two—so to speak—less interesting problems, that of establishing whether the two definitions are pre- or post-Euclidean, and that of determining if they involve propositions or figures. There is no doubt, in fact, that Aristotle speaks of geometrical analysis as a known procedure, fitting perfectly with the Euclidean definitions. Trying to date the latter becomes, therefore, completely uninteresting. However, I would not go so far as Árpád Szábo [Szabó 1953, 378–389] and [Szabó 1974] and attribute the discovery of analysis to the Eleates, claiming its derivation from Zeno's *reductio ad absurdum*. It is certainly true that analysis, when it deduces a false consequence from a certain hypothesis, falsifies that hypothesis, but it is not equally true that the Eleatic's arguments, especially Zeno's famous paradoxes, are *reductio ad absurdum*. They are indeed dialectical demonstrations, as noted by Plato, who presents Zeno as a dialectician in the *Parmenides*, and by Aristotle, who referred to Zeno as the “inventor” of dialectics in his lost dialogue entitled the *Sophist*. In geometry, the derivation of the absurdity of a conclusion results from its contradiction with other premises, so that, in this context, *reductio ad absurdum* are always convertible to direct demonstrations. In dialectical demonstrations, as those attributed to Zeno, the absurdity of a conclusion results, instead, from its contradiction with itself and it does not need to refer to other premises [Berti 1987]. There is no doubt, moreover, that the procedure of analysis described by Aristotle in the *Posterior Analytics* concerns propositions, not figures. Only with regard to the formers, it is meaningful to speak of convertibility. This does not affect the fact that an analysis by means of figures also existed, consisting in the search for the solution of a problem by means of the construction of appropriate figures. This was also well known to Aristotle, as it is evident from two passages quoted above.

However, the problem of the “direction” of analysis is more interesting, i.e., whether it is a “way up” or a “way down”. A fundamental contribution to the solution of this question is given by the *Posterior Analytics*, which probably also helps to solve the first problem raised by the Euclidean definitions, that of the text.

The example given by Aristotle—in which it is impossible to prove the truth from the false and in which, consequently, all propositions are necessarily convertible—shows how the procedure followed by analysis is exactly the same to that followed by synthesis, which consists in the passage from premises to conclusions that follow necessarily from them (and it is generally called “deduction”). This is why analysis is reliable in mathematics and not in “discussions”: exactly because it performs the same kind of deduction performed by the synthesis. When the propositions employed are convertible, it is possible to provide the same reasoning in both directions, without the risk of failing the proof. When, on the other hand, the propositions are not

convertible, analysis can fail—it may take a course that cannot be provided in the opposite direction by the synthesis. Therefore, it may fail in its objective of finding the proof of a certain hypothesis.

At this point, it is no longer important to establish whether analysis is a “way up” or a “way down”, since it is identical in its process to synthesis. We may say that it is a “way up”—as its very name indicates⁹—because it looks for principles, and principles are generally supposed to be higher than what comes from them.

On the other hand, the procedure applied by analysis is a deduction, which is a “way down”, in the same way as synthetic reasoning. Hence, analysis and synthesis are simply two opposite directions along which the same process can be followed. Yet, the legitimacy of analysis, i.e., the certainty that it follows a process that can also be followed in the opposite direction, is conditioned by the fact that the propositions of which it is composed must be convertible.

This condition occurs above all in mathematics because there, most often, there is no difference regarding the type of process, between analysis and synthesis, in the sense that both are deductions of necessary consequences, i.e., “ways down”. In other fields, however, like the discussions concerning everyday life, or the disciplines not reducible to mathematics, like physics (as understood by Aristotle) and ethics, the above-mentioned condition does not exist. Analysis, in these contexts, is not entirely reliable since it does not have the character of necessity that is required by synthesis and, thus, it can be considered as a “way up”. This reasoning may also explain the discrepancy between the two definitions of analysis in manuscript tradition. The older text, reported in **P**, makes no distinction between the procedure of analysis and that of synthesis, probably because, belonging to a mathematical context, it refers to the use of analysis and synthesis between convertible propositions, which is frequent especially in mathematics, as we have seen. The more recent text reported by the other manuscripts makes instead a distinction between the two procedures, and it is due, probably, to a correction suggested by the possibility of using analysis between non-convertible propositions as well.

A further confirmation of the different meanings of analysis, depending on whether it is applied to convertible or non-convertible propositions, can be found in Pappus’s description of analysis [*PAC* II, 634.11–636.14]. In the first part he says that in the analysis, once the sought-after is posited as granted, one looks for “that from which it derives” and “then again for the antecedent of that”, until one arrives at something which “in order is a principle” [η τὰς ἐν ἀρχῇ]. This clearly alludes to analysis conceived as a “way up”, not to geometrical analysis in particular.

In the second part of Pappus’s description, however, there is an account of analysis that is more specifically mathematical, or geometrical. It distinguishes between analysis of a theorematic type, consisting in the search for the

9. For ἀνά means “up” and λύσις means “solution”.

principles from which a theorem can be proved, and analysis of a problematic type, consisting in the search for the solution of a problem. Here Pappus refers to analysis that takes place in conditions of convertibility,¹⁰ that is, analysis conceived as a “way down”, which is the one actually practised in the rest of his work. With regard to the first part of Pappus’s illustration of analysis, it is worth noting a detail that may have had some importance for the history of philosophy. The Renaissance Latin translation of Pappus’s work, carried out by Federico Comandino in 1589, at the very point where the Greek text says that analysis, once the sought-after is posited as granted, seeks “that from which it derives”, renders this expression as *quid ex hoc contingat*, which means “what derives from this”. While the Greek text speaks of the search for antecedents, and therefore understands analysis as “the way up”, the Renaissance translation speaks of the search for consequents, and therefore understands analysis as “the way down”.

It is impossible to determine whether the translator had a different text from the one at our disposal, or whether he corrected the text on his own initiative. At any rate, it is safe to say that modern philosophers, from Galilei to Descartes, who read Pappus’s text in Latin translation, were forced to understand it as the way down, that is, as a procedure applicable to convertible propositions. This may have contributed, for example, to lead Descartes, who considered analysis as the philosophical procedure *par excellence*, to conceive the creation as ordered in a deterministic way, i.e., based on necessary connections in both senses, and to expose Galilei, who considered analysis (which he called the “resolutive method”) as *the* procedure of physical sciences [Berti 1983a], to some serious misunderstandings of his worldview.

The lack of distinction between the two types of analysis, one between convertible propositions and one between non-convertible propositions, may in fact have contributed to Galilei’s condemnation. His demonstration of the motion of the earth (true conclusion) from the tides phenomenon (false premise or explanation) gave to Urban VIII the impression of limiting divine omnipotence, attributing to the cosmic order a character of absolute necessity, such as is found only in mathematical processes, where it is impossible to demonstrate the true from the false [Berti 1983b]. If this is true, it will be a further confirmation of the importance of Aristotle’s clarifications.

Abbreviations

EE: *Euclidis Elementa*. 5 vol. Leipzig: B. G. Teubner, 1969-1977.

iEE: *Procli Diadochi in primum Euclidis Elementorum librum commentarii*. Leipzig: B. G. Teubner 1873.

10. And, in fact, Pappus [PAC II, 636.5] use the expression “and proof is the converse of the analysis” [καὶ ἡ ἀπόδειξις ἀντίστροφος τῇ ἀναλύσει].

PAC: *Pappi Alexandrini Collectionis quae supersunt, e libris manu scriptis edidit latina interpretatione et commentariis instruxit F. Hulsch.* 3 vol. Berlin: Weidmann 1876–1878.

Bibliography

- AL-NAIRIZI [1899], *Anaritii in decem libros priores Elementorum Euclidis commentarii. Ex interpretatione Gherardi Cremonensis in codice Cracoviensi 569 servata*, Leipzig: B. G. Teubner, edidit M. Curtze; supplementum ad *Euclidis Opera Omnia*, ediderunt I. L. Heiberg et H. Menge. Leipzig, B. G. Teubner 1883-1916.
- BERTI, Enrico [1983a], Differenza tra il metodo risolutivo degli aristotelici e la “resolutio” dei matematici, in: *Aristotelismo veneto e scienza moderna*, edited by L. Olivieri, Padova: Antenore, vol. 1, 435–457.
- [1983b], Implicazioni filosofiche della condanna di Galilei, *Giornale di metafisica*, 5, 239–261.
- [1984], L’analisi geometrica della tradizione euclidea e l’analitica di Aristotele, in: *La scienza ellenistica*, edited by G. Giannantoni & M. Vegetti, Naples: Bibliopolis, 93–127.
- [1987], *Contraddizione e dialettica negli antichi e nei moderni*, Palermo: L’epos.
- BRETSCHNEIDER, Carl Anton [1870], *Die Geometrie und die Geometer vor Eukleides. Ein historischer Versuch*, Leipzig: B. G. Teubner.
- CAMBIANO, Giuseppe [1967], Il metodo ipotetico e le origini della sistemazione Euclidea della geometria, *Rivista di filosofia*, 58, 115–149.
- CORNFORD, Francis M. [1932], Mathematics and Dialectic in the Republic VI–VII.(I.), *Mind*, XLI(161), 37–52, doi: 10.1093/mind/XLI.161.37.
- GALILEO GALILEI [1842], *Le Opere di Galileo Galilei, prima edizione completa condotta sugli autentici manoscritti palatini e dedicata a S. A. I. e R. Leopoldo II Granduca di Toscana*, Florence: Società editrice fiorentina, 15 vols., ed. by E. Alberi.
- GULLEY, Norman [1958], Greek geometrical analysis, *Phronesis*, 3(1), 1–14, doi: 10.1163/156852858X00011.
- HEATH, Thomas L. [1908], *The Thirteen Books of the Euclid’s Elements*, Cambridge: Cambridge University Press, 2nd ed. reprinted by Dover, New York, 1956.

- [1921], *A History of Greek Mathematics*, vol. I: From Thales to Euclid, Oxford: Clarendon Press, 2nd edn.
- [1949], *Mathematics in Aristotle*, Oxford: Clarendon Press.
- HEIBERG, J. L. [1903], Paralipomena zu Euklid, *Hermes*, 38(1), 46–74, doi: 10.2307/4472878.
- HINTIKKA, Jaakko & REMES, Unto (eds.) [1974], *The Method of Analysis: Its Geometrical Origins and Its General Significance*, *Boston Studies in the Philosophy of Science*, vol. XXV, Dordrecht; Boston: Reidel.
- MAHONEY, Michael S. [1968], Another look at Greek geometrical analysis, *Archive for History of Exact Sciences*, 5(3), 318–348, doi: 10.1007/BF00411631.
- MEKLER, Siegfried [1902], *Academicorum Philosophorum Index Herculanensis*, Berlin: Weidmann.
- MILHAUD, Gaston [1934], *Les Philosophes géomètres de la Grèce*, Paris: Vrin.
- MUGLER, Charles [1948], *Platon et la recherche mathématique de son époque*, Strasbourg; Zurich: Éditions P. H. Heitz.
- ROBINSON, Richard [1936], Analysis in Greek geometry, *Mind*, 45(180), 464–473, doi: 10.1093/mind/XLV.180.464.
- SZABÓ, Árpád [1953], Towards a history of greek dialectics (zurgeschichte der griechischen dialektik), *Acta Antiqua Academiae Scientiarum Hungaricae I*, 377–410.
- [1974], Working backward and proving by synthesis, in: *The Method of Analysis: Its Geometrical Origins and Its General Significance*, edited by J. Hintikka & U. Remes, Dordrecht; Boston: Reidel, *Boston Studies in the Philosophy of Science*, vol. XXV, 118–130.
- TANNERY, Paul [1915], Du sens des mots analyse et synthèse chez les Grecs et de leur algèbre géométrique, *Mémoires scientifiques*, III, 162–169.