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Some arguments for propositional logic: MacColl as a philosopher

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Résumé : L'article examine les raisons philosophiques, plutôt que mathématiques ou logiques, pour lesquelles MacColl a pu vouloir développer une logique propositionnelle. Nous trouvons des éléments de réponse dans le refus discret d'endosser une logique des choses, dans l'anti-psychologisme nuancé de MacColl, et dans une subtile épistémologie de la certitude liée à un usage méthodique de la grammaire.

Abstract: The paper considers the philosophical, rather than mathematical or logical, reasons why MacColl decided to develop a propositional logic. We find some answers in a discrete refusal to countenance a logic of things, in MacColl's qualified anti-psychologism, and in a subtle epistemology of certainty linked to the methodical use of grammar.

Before the end of the 19th century, logic developed without any conscious need of a specific calculus of propositions. After more than twenty centuries of Aristotelian syllogistic, a formal system was eventually created whose atomic formulæ are propositional variables. Łukasiewicz emphasized Frege's decisive role in that birth:

[...S]uddenly, without any possible historical explanation, modern propositional logic springs with almost perfect completeness into the gifted mind of Gottlob Frege. [Łukasiewicz 1967, 84]

It is nowadays well known that it was developed quite independently in the late 1870s by MacColl [MacColl 1877], Frege [Frege 1879] and Peirce [Peirce 1880]. As for the absence of any possible historical explanation, it has also been qualified by the study of its sources, in Bolzano and Boole in particular. It generally results in the assumption that the death knell for Aristotelian logic was given for “technical” reasons, that is chiefly in order to allow more convenient procedures for calculus, especially in the wake of the development of algebras of logic.

The object of the present paper is to take a glimpse at some possible reasons MacColl considered for developing a logic of propositions without either an impulse from mathematics or by reason of logical requirements proper, but rather on a philosophical basis. “Philosophical” is here meant to include epistemology, metaphysics and psychology. Can MacColl seriously be held not only as a great logician but also as a great philosopher of logic?¹ To be sure, he advocated his calculus of equivalent statements as a mere mathematical tool for problems such as handling the limits of multiple integers. Nevertheless, given that MacColl aspired to produce essays “of a real philosophic depth” [MacColl 1910, 186], one would expect his reasons to be more profound than mere technical skills to propel the revolution of modern logic in which he took part. Propositional logic is not only a new calculus or a mere study of logical operators, it supposes a new, truth-preserving semantics, a concept of grammar, a clarification of such fundamental notions as inference and substitution, more generally a new approach of the mental, and perhaps even a philosophy of space and time. The search for such a proper philosophical rationale in MacColl’s writings is the aim of the present paper.

MacColl’s propositional calculus

Before investigating the philosophical motivations, if any, for MacColl’s logic of propositions, one must clarify its role and status in his general system of logic. The reflections about MacColl’s propositional logic, during his lifetime and ever since, have sometimes failed to distinguish various senses of propositional logic. The most obvious one is a logic of propositions as opposed to a logic of terms. It maps the distinction between classes (of things or events) and truth-value bearers. Propositional logic thus presents a more suitable alternative to Peripatetic logic.² But it can also refer to the axiomatic formalization of the propositional variables in terms of inference rules and truth-functions (Frege, Russell) as opposed to an implicit interpretation, called by Alonzo Church “the older point of view” (whose adherents included MacColl, Peirce, Schröder and to a certain extent Peano) [Church 1956, 157]. The axiomatic propositional calculus then opposes syllogistic logic with its intuitive interpretation. Furthermore, to the supporters of an algebra of logic, propositional logic can designate the precedence of equivalence over equality and the central position of the implication relation. It thus stands in contrast with the algebraic syntax of Boolean logic.³

1. At least John R. Spencer recognized MacColl as a philosopher of logic in [Spencer 1973].

2. Roughly speaking, propositional logic opposes Aristotelian logic, though the former corresponds more or less to the hypothetical syllogism in Aristotelian and medieval logic [Castrillo 1994].

3. Not to mention that a logic of propositions differs from a logic of judgments, the former only belonging to formal logic (cf. Bolzano).

One can argue whether or not MacColl had a propositional logic according not only to the previous contrasts, but to the importance given to the following criteria. There is first a question of chronological priority, which is no doubt minor and need not seriously distract historians. More interesting is the foundational problem of logical priority: does propositional logic provide a basis for a logic of classes, or does it go the other way round? The problem is that such a question is biased: looking for a foundation, one is likely to discover logicism as a solution which denies the logical primacy of propositions over sets. Hence, a third question is to be asked: is logic only a convenient tool, or does it reveal some truth about reality? For instance, does a logic based on events require an ontology of events? Does a logic of propositions need a philosophy of propositions?

In which of the previous senses did MacColl develop a propositional calculus? Whatever be the exact chronology of the articles published, he thought of himself as a pioneer:

Alone, or nearly so, among logicians, I have always held the opinion, and my recent studies have confirmed it, that the simplest and the most effective system of Symbolic Logic is that whose elementary constituent symbols denote—not classes, not properties, not numbers, ratios, regions, or magnitudes, not *things* of any kind—but complete statements. [MacColl 1899, 108–109]

In writing that he was *nearly* alone, MacColl may have had in mind his colleague Peirce, with whom he was long acquainted, and who, without mentioning Frege's *Begriffsschrift*, attributed to himself and MacColl alone the idea of making “the *elements* propositions or predicates and not common nouns” [Peirce 1906]. But in including predicates in his list, Peirce clearly misses part of the import of MacColl's innovation.⁴ Such seems to be Couturat's judgment: in presenting the principal systems of symbolic logic (including those of Jevons, Peirce, MacColl and Peano) as quite dissimilar, he makes that of MacColl alone consist “in considering propositions alone as the elements of reasoning, and in assigning to them three distinct values” [Couturat & Ladd-Franklin 1902, 645].

However, MacColl's and Peirce's systems share an important feature: they ground the whole of logic on propositional calculus. If an interpretation of MacColl's formal system in terms of classes is still possible, the calculus of statements is more basic. As for Peirce's alpha graphs, which are at least “isomorphic to the theory of propositional logic” [Pietarinen 2006, 348] in representing Peirce's “calculus of first intention,” they are prior to first-order predicate logic (represented by beta graphs): it proves their fundamental nature. It is noteworthy that Schröder reproached such “MacColl-Peirce'schen

4. Nevertheless, MacColl does not make a big distinction between the predicative form (A is B) and the adjectival form (the A which is B); that is, A_B assumes A^B [MacColl 1897, 495].

Aussagenlogik” for putting the cart before the horse, since for him propositional calculus is viewed as a specialization of his calculus of domains [Peckhaus 1998, 29]. It calls attention to the fact that, like Peirce, MacColl played a role in the birth of modern propositional logic not only because of its systematic development, but in constantly arguing for its fundamental place at the base of logic. Indeed, the rather solitary position of MacColl does not mean that other logicians ignored the calculus of propositions, but often rather that they explicitly rejected its relevance.⁵

A mere emphasis?

MacColl often affirms that letters should *only* represent statements. As early as 1878, he insists that with him “every single letter, as well as every combination of letters, always denotes a *statement*” [MacColl 1878, 27]. This early position nevertheless needs to be qualified in order to respond to some obvious objections: symbols (and especially letters) often stand for numbers, ratios, subjects or predicates. That letters represent propositions should not be a “cast-iron system of notation” [MacColl 1907a, 472]. Hence, it could appear that MacColl’s originality mainly rests upon the emphasis he repeatedly placed upon the need for a propositional interpretation, among others, of the algebraic calculus. In other words, his logic of propositions would be nothing but a defense of pluralism.

What is more, even in his early years MacColl still had an ambiguous position *vis-à-vis* propositional logic. Though he claims that the ultimate elements of his system are statements, he nevertheless treats inference as holding between sentences *or* more atomic terms. After noticing that the relation between subject and predicate is analogous to the relation between antecedent and consequent, he indeed analyses a syllogism as a complex implication whose premises are expressed in an implicational form. It maintains a confusion in identifying “the ‘physical’ combination of propositions into a system with the ‘chemical’ combination of subject and predication into a proposition” [Shearman 1906, 11]. In short, in 1880, inference is said to take place between statements, but is used between a subject and a predicate (or more precisely, between two attributions of predicates). It shows that MacColl did not from the start have a clear conception of the difference between first-order and “zeroth-order” logic.

A parting from Boolean algebra?

MacColl’s life shows a deep striking break: although he never stopped (re)publishing mathematical puzzles for *The Educational Times* and meanwhile produced two books of literature, he put his logical work to rest between 1883 or so and 1896. He referred to this temporary abdication from logic as

5. See [Ladd-Franklin 1883], [Mitchell 1883], [Schröder 1891], [Peano 1896-1897], [Shearman 1905]. For an opposite position, see [Johnson 1892].

a long pause in his mental life. MacColl himself stressed the difference of his new, 20th century views from his 1877-1883 period:

[T]he principle which underlies my method... did not appear (in my earlier papers) to lead to any essential difference in the symbolic processes. That this is no longer the case my recent papers in MIND and in the *Proceedings of the Mathematical Society* will show; but the new development is still further removed than the old from the allied algebras which it has been the great aim of Mr. Whitehead to unite into one general comprehensive system. [MacColl 1899, 109]

If his first system was accidentally analogous to Boole's logic, his second is deliberately free from any Boolean influence. As compared with the other systems, the outward "resemblances of mere form hide important differences in matter, method, and limits of application" [MacColl 1903, 364]. Propositional logic, MacColl claims, is not equivalent to equational logic, even if the algebraists "found that, *in all the numerous cases they had examined*, the formulæ of the logic of Pure Statements had their exact analogues in the logic of Concrete Quantity, and they inferred, very naturally but quite erroneously, that this must also hold in the infinite number of cases which they had not examined" [MacColl 1899, 111].

Jevons is one of those who argued for equations against propositions and explicitly criticized MacColl's choice:

Even his letter-terms differ in meaning from mine, since his letters denote propositions, not things. Thus $A : B$ asserts that the statement A implies the statement B , or that whenever A is true, B is also true... It is difficult to believe that there is any advantage in these innovations. [Jevons 1880, xv]

MacColl intends to replace the syntactic operator of equality with the grammatical notion of synonymy within language:

I think I may predict that *synonyms* are destined to play an important part in the future development of symbolic logic. [MacColl 1897, 500]

Two words, two propositions are not synonymous for all eternity, because signification depends on context:

As new needs and new ideas arise with the growth of civilisation and the general advance of humanity, do we not often find that words which were at first synonyms gradually differentiate and, while still remaining synonymous in some combinations, cease to be so in others? [MacColl 1897, 500]

That is why the relation of equality has too much strength and fixity. MacColl's recurrent principle, "perhaps the most important principle underlying [his] system of notation" [MacColl 1902, 362], that there is "nothing sacred or eternal about symbols" [MacColl 1906a, 1], can even be interpreted in the sense that there are no fundamental relations in logic:

I see no reason why we should treat our symbols of relation, +, :, =, indices, fractional forms, etc., with more respect than we do the ordinary letters of the alphabet. [MacColl 1899, 113]

Does he criticize simply our use of the graphical marks, or of the relations themselves? Perhaps the second alternative, for “as even the so-called fixed stars are only found to be *relatively* fixed, so our so-called *constant* symbols of relation need only be *relatively* constant” [MacColl 1897, 495]. The syntactic framework of algebra is too narrow for such a semantics.

A rejection of terms and things?

Jevons claims that his letters denote things, not propositions. It is that for which MacColl reproaches Boole. Boole is indeed mainly concerned with terms, which, treated extensionally, can be reduced to a logic of classes.⁶

MacColl probably did not want to rid logic of classes, which would be nonsense. Perhaps it would be more exact to say that in emphasizing the role of propositions, he in fact aspired to create a calculus of statements that would transcend the traditional partition of various logics. MacColl’s propositional logic possesses the advantage of homogeneity of matter; it “is *one simple homogeneous system* which comprises (either directly or as easy deductions), all the valid formulæ of their two divisions,” namely, “the logic of *class inclusion* and the logic of *propositions*” (as well as many other valid formulæ) [MacColl 1903, 355].

To a large extent, this opposition has been viewed as the necessary choice between placing either inclusion or implication at the heart of logic. Peirce is notoriously guilty of a careless shift of his “scorpion tail” operator from implication to inclusion, for he turned out to consider that transitivity always depends upon inclusion [Peirce 1897].⁷ MacColl was aware of such a conflation of class inclusion with implication, reproaching some logicians for “us[ing] their symbol $\prec$ in one sense in their logic of *class inclusion*, and in quite a different

6. Nevertheless, Boole writes:

Instead of classes of things, we shall have to substitute propositions, and for the relations of classes and individuals, we shall have to consider the connexions of propositions or of events. [Boole 1854, 162]

Boole was allegedly about to develop a viable logic of propositions in his early *Mathematical Analysis of Logic*, though the project remained uncompleted, and was replaced by another approach in the later *Laws of Thought* [Hailperin 1984]. On the other hand, MacColl sometimes tends to minimize the difference between class and statement: he acknowledges that AB is a statement while (AB) should be viewed as a class, inviting us not to give so much importance to this difference. In conclusion, the status of sentential logic as compared to class logic can be read as rather vacillating and not too strict.

7. At least Peirce intended to subject this single operator to multiple interpretations such as the copula, class inclusion or material implication. For a thorough study of this issue, see [Anellis 1995].

sense in their logic of *propositions*” [MacColl 1903, 356]. He insists that not only is this operator unnatural as a sign, but it “is not (virtually) equivalent to $A : B$, i. e. the statement A implies the statement B ” [MacColl 1906a, 78].⁸

As a consequence of replacing things by propositions, MacColl insists that “though this system does not necessarily exclude metaphysical considerations,” it is theoretically and practically independent of such considerations and is all the more reliable for that reason [MacColl 1880, 58]. It would probably be far-fetched to argue that MacColl wanted to get rid of the ontology of substance, as the pragmatist maxim would do: Peirce’s logic of propositions no doubt has deeper philosophical roots. But at least, no ontology is required by pure logic. Thus, one only has to investigate the relations between signs:

[...] we should, in my opinion, first investigate the mutual relations of these statements, representing each by its own independent symbol, and call this process of investigation Pure Logic. [MacColl 1899, 109]

“The logic of functions or relations” is mentioned in MacColl’s fourth “Symbolic reasoning” article in 1902. He there writes that the logic of relations does not belong to the “logic of statements” but is a development of it in a particular direction, exactly in the same way as the theory of functions is a particular development of pure mathematics.

For this reason, in order to mark the analogy, the *logic of relations* should rather be called the *logic of functions*. [MacColl 1902, 362]

If MacColl does not consider the function-theoretical approach as pure logic, it is because he fails to see the interest of a logic of relations, and to grasp the concept of a function that would not depend on the theory of language. Hence the failure to give to propositional functions the place they deserve in logic.⁹

8. For instance $\eta : \varepsilon$ is a formal certainty, $\eta \prec \varepsilon$ a formal impossibility. Another example is that $(A : B) (B : C) : (A : C)$ makes sense, in contrast to $(A \prec B) (B \prec C) \prec (A \prec C)$.

9. Indeed, MacColl wrote:

What Mr. Russell calls a *propositional function*, I should prefer calling a *functional proposition*; but whatever locution we adopt, it must, from my point of view, be classed as a proposition. [MacColl 1907a, 470]

Shearman criticised this position as follows:

Mr. MacColl allows his symbols indiscriminately to represent propositions and propositional functions; but, in so far as he has done so, he has not assisted in producing the newer Symbolic Logic, for in this it is a matter of fundamental importance to draw a clear line of distinction between the two uses. [Shearman 1906, 151–152].

An investigation of the laws of mind

The laws of thought

One of the reasons for adopting a logic of propositions rather than of predicates paradoxically seems to lie in a certain conception of the mental. Of course MacColl defends an unpsychological view of logic. He argues “that formal logic should not be mixed up with psychology—that its formulæ are independent of the varying mental attitudes of individuals” [MacColl 1906a, 60]. And his formal system also shows evidences of anti-psychologism (for instance in embracing the paradoxes of strict implication). Meanwhile, MacColl is in search of the “fundamental rules of our ordinary reasoning” [MacColl 1880, 48], and defines (symbolic) logic as “the science of reasoning by the aid of representative symbols” [MacColl 1897, 493]. But surprisingly, when one remembers the psychological quest for man’s intellectual powers which motivated Boole’s logical works, the “second” MacColl, while harshly discarding the attempts of the Booleans to perfect an algebra of logic, extols Boole’s *philosophical* insights.

No one can admire Boole’s *Laws of Thought* more than I do. As a philosophical and speculative work it is brimful of profound thought and original suggestions, while its style is charmingly lucid and attractive. [MacColl 1897, 504]

MacColl claims Boole’s application of algebra to probability did not succeed, though his attempt has a philosophical interest. One can compare Boole and Shakespeare:

Both authors possessed a remarkable *analytical* insight into the workings of the human mind; the one of its secret motives and passions; the other of the subtle laws of its intellectual operations; yet both—the one judged by his plays, the other by his *Laws of Thought*—showed but little *constructive* ability. [MacColl 1897, 505]

Boole’s mistake was to endeavour to make the multiplicity of human thought fit into the given rules of algebra, “to squeeze all reasoning into the old cast-iron formulæ constructed specially for numbers and quantities” [MacColl 1897, 505]. Boole indeed alleges that there is an exact adequacy in the laws which conduct the operations of reasoning and of algebra [Boole 1854, 6]. In other words, the actual reasoning does not bear on symbols, be they mathematical or, as we will see, thought-signs; but the major fact Boole clearly understood is that it relies on transitivity.

Paradoxically, while dismissing psychologism, MacColl grants that an inquiry in the depths of logic nevertheless requires an acknowledgement of our mental processes—of Peirce’s so-called “anthropological facts” which “have a great bearing upon logic” [Peirce 1982, 362]:

Though in purely formal or symbolic logic it is generally best to avoid, when possible, all psychological considerations, yet these cannot be wholly thrust aside when we come to the close discussion of first principles, and of the exact meanings of the terms we use. [MacColl 1906a, 82]

It goes with MacColl's interest for more psychical and psychological matters at the end of his life. It would be erroneous to believe, MacColl writes, "that formal logic is absolutely independent of psychology—that they have nothing to do with each other", for mind is a "pre-condition" of a working logical or mathematical formula, be it human or "superhuman" [MacColl 1910, 191]—a kind of "cosmical reason," for Peirce. Such an apparent contradiction in MacColl is due both to a chronological evolution and, from the start, to his qualified and subtle anti-psychologism.

Inference vs. implication

Contrary to what the Russellian posterity could make one expect, the debate over propositional logic did not focus on the inference vs. inclusion opposition as much as on an inference vs. implication contrast at the time MacColl wrote. The priority of the concept is a postulate that separates Boole from some of his most important followers [Vassallo 1995, 89–91]. The former considered that a logic of terms should be first because the analysis of the laws which govern the conception should precede the analysis of those governing judgment and reasoning. It corresponds to the first step of the threefold division of logic that has been in use ever since Aquinas at least, namely a doctrine of terms related to the concept, a doctrine of propositions related to judgment, and a doctrine of inferences or syllogism related to reasoning.

Without going into MacColl's technical notion of implication,¹⁰ suffice it to recall his "law of implication," according to which "the sole function of the reason is to evolve fresh knowledge from the antecedent knowledge already laid up in the store-house of the memory" [MacColl 1880, 52]. It echoes Peirce's 1864 assumption that every judgment is an inference, since it relates something known to the unknown—save for the Kantian "I think" [Peirce 1982, 152]. Nevertheless, many logicians insisted that in contrast to ampliative reasoning, deduction does not lead from a known truth to a truth unknown, because its conclusion is a mere clarification of what was already admitted in the premises [Peirce 1982, 152]. Yet, MacColl paraphrases John Stuart Mill in writing that

10. In particular the distinction between logical and causal implication, the cases in which $\alpha + \beta'$ are not synonymous with α^β , the paradoxes of material implication, etc. Neither will we discuss the existential import of propositions, that is, the difference between $A : B$ and $A \therefore B$, which justifies the conviction that "*Not one syllogism* of the traditional logic—neither Darapti, nor Barbara, nor any other—is valid in the form in which it is usually presented in our text-books, and in which, I believe, it has been always presented ever since the time of Aristotle" [MacColl 1906a, 47].

“[w]e must reason from the known to the unknown whatever be the subject of investigation” [MacColl 1880, 58].

That is probably why thirty years later he would emphasize that implication is not inference, for “[a]s commonly understood, *inference* involves psychological considerations; *implication* does not” [MacColl 1910, 191]. Inference is about obtaining knowledge, whereas implication refers to the impossibility that a certain premise does not lead to a certain conclusion. It is noteworthy that, to him, symbolization reflects the inferential process, not the actual reasoning. Writing with symbols (and the logical relation of the truth of premisses to the truth of conclusion) does not accurately represent the process of implication. The words *if* and *therefore* are examples of terms for which we need psychological considerations [MacColl 1906a, 82]. And of course “it is convenient in scientific researches to consider the two as far as possible apart” [MacColl 1910, 191].

Inference is a temporal relation. Thus, contrary to Boole, MacColl refuses to view logic as concerned with temporal portions. It is a fundamental reason why MacColl denies that Boole developed a propositional logic or any calculus of pure statements. Its elementary constituents indeed never represent statements, “they always represent the fractions of time (referred to some understood arbitrary whole unit) during which the various statements in question are true; and it is quite clear from Boole’s language that he considered this convention as a necessary and fundamental principle” [MacColl 1899, 111]. *The Laws of Thought* indeed vacillates between two interpretations of signs, either representative of things or of operations of the human intellect. It in both cases should refer to the laws of reasoning holding for the “time for which the proposition is true.” Thus, Boole’s embryonic propositional logic is based on time [Godart-Wendling 2000].

It is reminiscent of the Stoics, who, though they developed a kind of propositional logic in a very different sense from what we mean, nevertheless did not ground their logic on the Aristotelian *apophansis* (the subject-predicate structure) but on the *axioma*, that is, a proposition stating events. Therefore, whereas Aristotelian reasoning is a system of inclusion of concepts, reasoning for Chrysippus bears on implications of temporal relations. MacColl’s rejection of time as a fundamental conception of logic shows the modernity of his notion of proposition. As a comparison, Peirce puts it that “a succession of time among ideas is obviously presupposed in the conception of a logical mind” [Peirce 1986, 106]. It goes with the Stoic idea that “[t]he letters must denote the objects of logical thought; that is facts or supposed facts” [Peirce 1866].

Logic and mathematics

Neither is logic concerned with parts of space. Not only is the calculus of pure statements not a set of mathematical operations, but it differs from

algebra as much as algebra differs from geometry [MacColl 1899, 111]. For it is an error to believe that:

[T]o every valid formula in Symbolic Logic, no matter how abstract the region of thought, and no matter what the elementary symbols may be defined as representing, there always corresponds an analogous formula (*symbolically* identical) of which the elementary symbols represent spatial magnitudes; so that any argument referring to the non-spatial region of thought has its symbolic *double* (so to speak) in the spatial region, and *vice versâ* [MacColl 1899, 110–111].

MacColl's assertion very nearly rejects Peirce's diagrammatical approach to reasoning. Statements, MacColl claims, have *sui generis* characters, which cannot be reduced to mathematical properties. He defends a linguistic conception of logic, completely at odds with Peirce's system of existential graphs, which "is essentially different from language" [Peirce 1910] and "expresses everything with a precision that no human tongue can approach" [Peirce 1905]. On the contrary, MacColl assumes that "the great advantage of this 'Calculus of Limits' is that it is *independent of all diagrams*" [MacColl 1906a, 140].

In short, MacColl views thought as conveyed by propositional entities that have no proper duration nor any situation or location. Having refused to ground logic in time or space, MacColl cannot say that logic is a branch either of arithmetic or of geometry. Propositional logic is not mathematical, as Peirce would perhaps have held when stating that mathematics is logic rather than that logic is mathematics [Peirce 1897].¹¹ MacColl refuses to trust some more or less superficial or fancied analogies between logic and mathematics. The "law of symbolic coincidence" between pure (that is, propositional) logic and algebra should no longer be relied on. But on the other hand, MacColl does not draw such a sharp line between his mathematical puzzles and logic, not afraid, as was Peirce, that logic "is in danger of degenerating into a mathematical recreation" [Peirce 1883, 143]. MacColl's main interest indeed lies in what Peirce calls the *logica utens*, that is, logic as a tool, especially for problems of probability and limits. Peirce draws the attention of his fellow logician to this difference:

[...] my studies in symbolic logic have differed from yours in that my aim has not been to apply the system to the working out of problems, as yours has, but to aid in the study of logic itself.
[Peirce 1906]

MacColl in fact states that the question of the primacy of logic or of mathematics over the other is pointless, for they do not have the same domain: mathematics deals with things, logic with signs. The question of which science subsumes which is meaningless, argues MacColl, who "would like to contribute

11. We of course do not intend to discuss Peirce's relation to logicism. On this matter, see e.g. [Houser 1993].

his humble share as a peacemaker between the two sciences, both of which he profoundly respects and admires" [MacColl 1880, 47].

An investigation of the laws of signification

A logic of statements or of propositions?

Whereas mathematics works on things and human thought processes ideas, symbolic logic deals with signs. MacColl has no inclination for mental symbolics, for "[i]n *mental* reasoning we dispense entirely with symbols" [MacColl 1880, 48]. It goes with an explicit rejection [MacColl 1910, 350] of William James's thesis that "the thoughts themselves are the thinkers" [James 1892, 83], and also boldly contradicts Peirce's famous thesis that "every species of actual cognition is of the nature of a sign" [Peirce 1986, 76].

MacColl needs a definition of the basic symbol, namely the statement. To this end, a basic theory of signs and language founded on natural evolution is developed. He does not precisely define a sign, and seems to identify it with what he calls a symbol. The examples he unvaryingly gives are sounds like "the 'caw' of a sentinel rook" or visual symbols like "the national flag of a passing ship." When a sign is likely to bear a meaning, it can be interpreted as a statement. A statement is what Peirce would simply call a sign, the indication of intelligible information or of a possible meaning for the logician. What meaning do they convey?

What does the "Caw" of the sentinel rook perched on the branch of a commanding tree say to the others on the ground busily feeding on the farmer's property? To one of these it may say, "A man is coming with a gun"; to another it may say, "A boy is coming with a catapult"; to all it says, "Danger approaches," though their respective ideas as to the precise danger may be vague and varied. [MacColl 1902, 365]

In other words, every single interpretation of a given sign describes a particular "immediate object," as Peirce calls it; immediate objects are fated to converge into the final, real object. The rook's "caw" refers to a particular fact, but its meaning is indeterminate, that is, vague and general. That is why it would be a mistake to reserve the name "propositions" to those which convey a real, determinate meaning, as opposed to propositional forms [MacColl 1910, 192].

As a statement is any sign which communicates information to ear or eye, it need not have a specific form. A statement which, *in regard to form*, has such a subject-predicate structure, is called a proposition.¹² The threefold partition of MacColl's classification of propositions puzzled many logicians:

12. In fact, MacColl's use is hardly rigorous. Furthermore, it gets blurred by the author's bilingualism: under MacColl's pen, the French word "*proposition*" often stands for the English "statement".

how can a proposition be a variable, that is, sometimes true and sometimes false? Here is MacColl's reply:

I do not say that the same *information* may be sometimes true and sometimes false, nor that the same *judgment* may be sometimes true and sometimes false; I only say that the same *proposition*—the same *form of words*—is sometimes true and sometimes false.
[MacColl 1907a, 470]

Thus, a proposition is simply a conventional arrangement of words or other symbols employed to convey information or express a judgment. In this respect, MacColl disagrees with Russell, who “does not consider a propositional form of words, such as ‘He is a doctor,’ a real proposition till it is actually employed to give information” [MacColl 1908, 453].

Nevertheless, the apparent simplicity of the concept of proposition still hides a major difficulty. For if in pure logic statements are represented by single letters, then a pure statement is, for instance, S , or P , that is, a subject (e.g. the national flag), or a predicate (e.g. being blue, white and red). In consequence, the logic based on a letter standing for “ S is P ” is not pure, although it is what *we* mean by propositional logic. In the same spirit, while MacColl asserts that a pure proposition is represented by a single letter, that is, is an indecomposable, atomic sign, he also explains that “ A^B is a pure proposition,” as opposed to being a modal proposition [MacColl 1906a, 94]. In other words, included among the statements are the terms of a proposition, so that the calculus of equivalent statements should be both a logic of classes and a logic of propositions so as to exclude the possibility of the calculus dealing with classes. MacColl's logical pluralism turns out to be rather embarrassing.

Logical classification and grammatical clarification

For MacColl, the task of logic is to classify statements according to attributes as true, false, certain, impossible, variable. Pure logic is just a classification of propositions:

[W]hatever be the subject of research, in order to articulate itself all reasoning requires propositions. Hence, in order to make our reasoning perfectly general and our formulæ universally applicable, the first goal of our research must be the classification of the different kinds of propositions and the relationships between them, and we have to call this work Pure Logic. [MacColl 1901, 135]¹³

MacColl writes that the first step in his classification is to specify the difference between A and “ A is true” [MacColl 1906b, 513]: A^τ is of a “higher degree” than

13. Quoted from the English translation in [Astroh 1998, 147].

A [MacColl 1906a, 17].¹⁴ To be true, truth values are not always clearly distinguished from modalities, predicates and sets of propositions. For instance, ε is both a modal predicate of necessity and a class statement, namely the class of all the necessary propositions. MacColl nevertheless lays the foundations for a hierarchical typology of propositions based on their order of predication (A^B being a pure proposition, A^{BC} a modal proposition of the first order, A^{BCD} of the second order, etc.).

He even lays the basis of an epistemic logic when suggesting that his three-dimensional system of logic (including the values of necessity, impossibility and uncertainty) could be completed with a “new and more subjective scheme” [MacColl 1897, 508]. Such values as “known to be true,” “known to be false” and “doubtful” are intended to better fit the ordinary facts of our consciousness and experience. Unfortunately, MacColl refuses to consider it seriously because he fears they might raise psychological questions. He would later even consider that the modality of certainty itself has an epistemic value: A^τ is said to be a revision and confirmation of A [MacColl 1906b, 515], which seems to explain logical necessity in terms of revision of beliefs, a dangerous method. The division of statements into certainties, impossibilities and variables should be “wholly independent of psychology” [MacColl 1907a, 473], though logic should indeed be concerned with ideas of certainty, not only with the true/false dichotomy. “All such attempts to surround symbolic logic by a Chinese wall of exclusion are futile” [MacColl 1906b, 515]. This difficulty is the reason why A. T. Shearman assumes that MacColl in fact quits the domain of pure logic when developing his modal logic:

Pure Logic can take account of the uncertainties that such data occasion, but the propositions dealt with will then denote not the relation of the respective letters to x , but the relation of the thinker to each implication. [Shearman 1906, 30]

MacColl’s originality in this discussion derives from the fact that he does not regard the enterprise of clarification as logical but linguistic. As evidence, the difference between “it rains” and “it is true that it rains” is present within our natural tongues. Grammar is concerned with the relations within a proposition, logic with the relations between propositions: that is why a logic of predicates is not a real logic. Logic could focus on the links between propositions rather than on the content of every proposition. The clarification of ordinary language through grammar is thus requisite for propositional logic. Grammar in this sense is not reduced to the meaning of words in a sentence but implies a whole context:

The mere word, and sometimes the complete grammatical proposition, when considered apart from context, may be meaningless or misleading. [MacColl 1907b, 162]

14. It corrects the previous assertion that “[t]he symbol A is short for A^τ , and may be read ‘ A is true’; the exponent τ being often left understood, like the sign $+$ in common algebra” [MacColl 1900, 75].

MacColl's philosophical insights are modest. His defense of propositional logic remains ambiguous: his attempt at reduction to one formalism conflicts with his hatred for dogmatic symbolization; his claim that symbols are only a question of economy [MacColl 1897, 493]¹⁵ contradicts the fundamental role of propositions; and his mathematical use of logic does not accord with the rejection of an algebraic syntax. Perhaps his best achievement does not consist so much in his logical reduction to a symbolic system of propositions as in his conviction that propositions are the true logical atoms—which would open the road to the traditional problems of logical atomism, such as the unity, objectivity and bipolarity of propositions. Such a view can nevertheless be said to have philosophical motivations: a discrete refusal to countenance a “meta-physical” logic of things, his qualified anti-psychologism (nonetheless refusing, like Frege and contrary to Peirce, to ground logic in the forms of the understanding), and a subtle epistemology of certainty linked to the methodical use of grammar, are the basic traits of MacColl's “philosophy” of logic.

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15. A weak argument for propositions, for one can easily argue that a logic of classes is more convenient. Sherman, for instance, writes:

As regards the question of economy of space in the solution of problems, the evidence seems to show that the class interpretation is to be preferred. [Shearman 1906, 21].

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