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Psychologism and the Cognitive Foundations of Mathematics

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1 Introduction

In this paper, I will attempt to purport the two following claims:

Claim1: Naturalistic theories of the cognition of Mathematics imply psychologism, unless one recognises the social nature of Mathematics.

Claim2: A non psychologistic enquiry into the cognitive foundations of mathematics should aim at disentangling the social and the cognitive causes at work in the making of mathematical knowledge.

My argument unfolds accordingly: In the first part of the paper, I analyse how an enquiry into the cognitive foundations of mathematics can turn out psychologistic, *i.e.*, assert that the truths of mathematics and logic are psychological facts. I then attempt to show what philosophical presumptions on the nature of mathematics lead to psychologism. I argue that the only way out of this *a poria* is to acknowledge that social normativity is an essential constituent of mathematical practice. In the last part, I briefly sketch what could be a non-psychologistic enquiry into the cognitive foundations of mathematics. I argue that such an enquiry should include — apart from psychological studies — (1) an anthropological analysis of the role of psychological facts in mathematical

practice and (2) a historical analysis of the cognitive constraints on the development of mathematical knowledge. Along these lines, I support the idea that Mathematics is at the same time a cognitive product and a cultural object. Thus, the theoretical position I advocate is, using the current labels, cognitivism plus sociologism; where cognitivism, for my purpose, is a research program that led to assert that there are specific competencies that underlay our knowledge of Mathematics and Logic, and sociologism is the theoretical position that asserts that knowledge is socially constituted and that leads to the study of the social texture of knowledge, such as scientific institutions, processes of communication, deference and, more importantly for my current concern, the normative aspects of knowledge.

2 Psychologism is coming back!

Psychologism, in its crude form, is the doctrine that asserts that “logic is a study of the mind” (Macnamara, 1986:10). This doctrine has been fought against and was officially dismissed by the arguments of Frege (1884, 1893 and 1894) and Husserl (1900). Their main argument was that the truths of logic are objective and independent of psychological empirical and subjective facts. Psychology deals with what people believe to be true while logic deals with what is necessarily true. Since Frege and Husserl the question seemed to be settled. I will argue, however, that psychologism is still a lively philosophical problem. To begin with, one of the present-day avatars of psychologism is the use of logic in psychology. Let us start with the work of cognitive science on rational behaviour.

Cognitive science asserts that human behaviour stems from, and can be accounted for by, cognitive processes. Applied to Mathematics, this means that the production of proofs and mathematical concepts should be explained in terms of cognitive processes. Moreover, one of the most important paradigms in cognitive science asserts that cognitive events are performances rendered possible thanks to some cognitive competences. This paradigm was first initiated by Chomsky’s theory of linguistic competence. The modular theory, first enounced by Fodor, specifies the functioning of some cognitive competencies, which are described as mental devices, or modules, which perform specific tasks. Modules are cognitive organs; their function is to perform specified computations on mental representations. Within this framework, it is natural and fruitful to hypothesise the existence of cognitive competencies such as a ‘logic

module' and/or an 'arithmetical module', which are mental devices that produce/perform logic and/or arithmetic. Yet, once this assumption is made, the threat of psychologism is not far. Here is the path that leads to it:

- (1) Producing Mathematics is using our cognitive mental devices.**
- (2) Thus Mathematics is the product of mental devices.**
- (3) Mathematics depends on the mental, its truth and content are psychological facts.**

Let me illustrate my argument with an analysis of Macnamara's research program. Macnamara was a leader cognitive psychologist who studied the cognitive foundations of reasoning while at the same time being conscious about the problem of psychologism, so I think he is a good representative of the theories that I intend to criticise. In "A Border dispute" (1986) Macnamara called for a research program based on the idea that the mind contains some innate devices from which originate our reasoning and that that constitute our basic logical skills. The goal of the program was therefore to discover those devices. He furthermore claimed that those devices essentially amount to some "basic logical skills". As a consequence, logic is the appropriate, and even essential, mathematical tool for the psychology of reasoning. This call was taken seriously, for in 1994, Macnamara edited, together with the G. Reyes, a book called "The Logical Foundation of Cognition" containing articles responding to Macnamara's program with the logic of types. Our basic logical skills, according to Macnamara, constitute a "mental logic", which accounts for our linguistic resources of expression and understanding and our ability to grasp inference. More generally, "The mind in part of its functioning applies the principles of that [mental] logic". The mental logic is in correspondence with "each ideal logic (true to intuition)" [Macnamara 1986, 22] and includes fundamental principles such as the principle of contradiction. Logical competence is error free and "gives rise to intuition of absolute necessity" [Macnamara 1986, 28]. It constitutes a 'competence', as opposed to 'performance', following Chomsky's distinction in his theory of universal grammar. That is to say that the mental logic, or logical competence, is not framing all our thoughts as in a Kantian theory (sometimes called transcendental psychologism). The mental logic constitutes an aptitude that we can, and must, call on in order to perform good reasoning. The main purpose of this distinction is to allow the possibility of logical errors in the performance of logical tasks. In that way Macnamara wants to account for the intuitions that:

- The layman has the same ideals (in his behaviour) as logic (e.g. consistency).
- The building of formal logic is based /founded on basic logical intuitions.
- The learning of formal logic is necessarily based on basic logical intuitions.

The logical competence “abstracts from logical error, from other psychological functioning that accompanies logical thought, and from the specifics of the many devices that could apply the competence” [Macnamara 1986, 27].

Macnamara defends his theory against the accusation of being psychologistic. His claims, he says, “have to do with access to logical principles, not with justifying them” (1986:42), the latter being the work of logicians. But what does Macnamara mean by “access to logical principles”? It seems that mental logic is a kind of ladder which gives access to the objective realm of logic. In that case the truth is already there; mathematicians describe it and psychologists describe how and why the description is possible. Macnamara, however, explains the human possibility of doing logic with the basic principles of logic. When he considers our access to logical connectors he merely asserts that logical connectors are already in our minds. The use we make of connectors is the result of the activation of our logical competence, the mental logic. Hence the description of the access is a generation of the truths of logic plus the assertion that this generation stems from psychological facts. The ladder is exactly our making of logic. Still, Macnamara continues his defence by softening the meaning of ‘access to logical principles’. Mental logic, he says, does not generate logic; it only assesses its validity. So, let us consider how the assessment procedure works. If the basic logical competence is just the cause of our conviction, then the assertion is just that we have a feeling of certainty because we have an innate feeling of certainty. This does not provide, as Macnamara claims, “the key element in the psychology of human reasoning”. So the assessment procedure includes some generative consequences, and in particular, the making of logic. It is as if we produced some logical-like propositions, and only the truly logical ones were passing the tests. So here again logic finds its justification in psychological facts, namely passing the assessment tests of mental logic. Logic is in Macnamara’s theory, the very result of our (ideal) performing of the logical competence. The truths and the laws of logic can be reduced to laws of psychology because the formers are just the expression of some characteristics of our mind. But such characterisations of our mind are actually laws of psychology. In brief, to give a

mental reality to the laws of logic implies that the objectivity and the normative character of logic stem from the laws of thought. Eventually, a last argument of Macnamara consists in appealing to Platonism: the semantics of mathematics stems from referring to the Platonistic world of mathematical entities. Yet, the knowledge of mathematics is possible through *mental* logic. Macnamara's epistemology does not include an account of how we refer to, or have any intuition of, Platonistic entities *outside* our minds. On the contrary, his cognitive account of logical knowledge is wholly **internalist**. (The Platonistic School has, however, provided an account of how reference to Platonist entities is possible. So I will come back to it shortly.)

Macnamara's theory is a moderate version of psychologism because it asserts that logic expresses properties of the mind but not in the sense of Mill that "logic is an introspective science generalising over inferences that are judged necessary" [Macnamara 1986, 10]. Logic is not an empirical science generalising over people's reasoning. It is psychology that must contain logic as an *a priori* science. Thus Macnamara avoids some of the strong criticisms of Frege and Husserl against psychologism by coming back to a theory that looks very much like Kant's 'transcendental psychology'. Meanwhile, his competence/performance distinction allows him to avoid the difficulties of a strict kantian theory with regard to the possibility of logical error. Yet, Macnamara's psychologism becomes radical when we add, as Macnamara does, that the autonomy of logic with regard to performance, far from bringing us afar from the laws of psychology is precisely what allows us to really grasp the logical competence, that is to say without the disturbance of the performance. In this context, the role of the psychologists of reasoning is to find a system of logic that is appropriate to encompass both our day-to-day reasoning and mathematical logic; and the role of logicians is to develop that system. With this share of the tasks that Macnamara seems to assume, one cannot accuse him of saying that logic is a study of the mind. It remains however that logic is expressing some laws of psychology. The truths of logic are psychological facts. Macnamara deludes himself when he thinks that (his) psychology can be neutral with regard to the ontology of logic. Macnamara defines human rationality as essentially being the product of innate logical abilities. In doing so, he reduces logic to the expression of the functioning of human rationality, which in the last analysis is psychologism.

Recently, other mathematical competencies have been posited by psychologists. The cognitive foundations of arithmetic have in particular been the object of interesting experiments and theories. Based on ani-

mals and pre-linguistic children ability to distinguish different quantities, psychologists have asserted the existence of a ‘real number system in the brain’ [Dehaene 1996] [Gallistel & Gelman, 2000].

Of course, not every psychologist is convinced of the existence of a logical module and other specific mathematical competencies (*e.g.*, Johnson-Laird, see [Johnson-Laird & Byrne 1991]). My present concern, however, bears not so much on the specific capacities psychologists posit, than on the relation between these capacities and Mathematics. As a matter of fact, I believe that these psychological investigations and theories go in the right direction. Yet, for those theories not to be disconfirmed by the arguments already raised by Frege and Husserl against psychologism, the role of these capacities in the making of mathematics must be clarified. While Dehaene’s assertions seems wise and modest: he urges teachers of mathematics not to discard basic intuitions in their teaching on the basis that such mathematical intuitions do exist and should play a role when learning mathematics, others have made stronger and rather bold assertions. Thus, the logician Krivine, supported by the philosopher J. Petitot, asserts in a popular French science journal [Science et Vie February 2002], - the heading, for the occasion ‘Intelligence reveals its true nature’), that mathematical theorems are nothing but discoveries on the functioning of our mind. Godel’s first incompleteness theorem, for example, would be the mathematical discovery of our sleeping program. The psychologists Gallistel, Gelman and Cordes have a somewhat similar view. In a paper whose title is “The cultural and Evolutionary History of the Real Numbers” they say: “Our thesis is that [the] cultural creation of the real number was a platonistic rediscovery of the underlying non-verbal system of arithmetic reasoning. The cultural history of the real number concept is the history of our learning to talk coherently about a system of reasoning with real numbers that predates our ability to talk, both phylogenetically and ontogenetically”. Of course, if the paper successfully shows that “there exists a common system for representing both countable and uncountable quantity by means of mental magnitudes formally equivalent to real numbers”, it actually says nothing about the cultural history or how the ‘platonistic rediscovery’ happened. To finish with a philosopher, Penelope Maddy asserts that we “possess intuitive, non-linguistic knowledge of general facts about sets, and intuitive principles like the simpler axioms and the iterative conception are justified by their accuracy in formulating this intuitive knowledge” (1980: 189). In order to save Godel’s Platonism from the epistemologist’s criticism \tilde{U} how do we access the Platonic world of mathematical entities? -, she describes

mathematical intuition in terms of neurophysiological processes (1990). Maddy, here, is very close to psychologistic theories. Yet, with Godel, she asserts that Mathematical concepts do refer to things outside our minds, which are (contrary to Godel and in accordance with Quine and Putnam) in the physical world. This, if it was true, should save her from psychologism. Unfortunately, as she acknowledges herself later (1996) (The later ‘naturalist Maddy’ criticises the previous ‘realist Maddy’, in this paper, I normally refer to the realist Maddy), this does not square with the practice of mathematics: mathematicians do not intend to refer to things in the physical world and, more importantly, they do not call on how the world is to justify their claim. It is, however, another assertion that actually makes Maddy’s account not psychologistic. She says: “[mathematical] intuitions can be false, so no matter how obvious they seem, they must be confirmed like any theory, and like any theory, they can be overthrown”. Intuitions do play a major role in mathematics, but they do not have the last word. This immunises Maddy’s theory from the criticism against psychologism but let us with another problem: if mathematical intuitions are not sufficient for assessing the truth of mathematics, what is?

3 Where is then the norm? Strategies for avoiding psychologism

The kernel of the refutation of psychologism, I believe, relies on the essentially normative component of mathematical practice: In mathematics, we just cannot say whatever we want, even if it corresponds to our individual strongest intuition. The problem with psychologism, is that it cannot account for this fact. In the cognitivist setting, this means that mathematical concepts or proof procedures cannot designate a class of mental processes that would necessarily produce mathematical truths. At most, cognitive processes sustain and allow definitions to have some content and proof procedures to be applied. Mathematics is normative and norms differ from competence:

1. A standard of justification is something external. It is not subjective.
2. Justifications are performance. It is these performances that are the object of assessment (truth, coherence, . . .), not the underlying competence.
3. We cannot give an instance of the right cognitive ability which is to be used when doing mathematics. But even if this was possible,

doing mathematics would not consist in categorising neurological events.

On the other hand, we do want to give a role to mathematical intuition. Mathematicians are cognitive systems; they use their minds when doing mathematics. The appeal to basic mathematical intuitions acknowledges this fact and points out the necessity to rely, in the last instance, to irreducible basic cognitive processes. At some point, formal justification must end and give place to a mere feeling of certainty. Even if the mathematician's job is not to analyse or even talk about intuitions, it remains that he uses these intuitions to do Mathematics.

There are various strategies for recognising the normative aspects of mathematics, and thus avoiding psychologism, and at the same time giving a proper role to cognitive processes. Contemporary Platonism, Modesty and the cognitively informed Sociologism that I will advocate, are such strategies. Formalism, however, is not, since it denies as much as possible the role of intuitions. Intuitionism and logicism, or set-theorism, acknowledge the role of only a restricted portion of the intuitions at work in mathematical practice. So I remain with the task of showing that my strategy is better than Platonism and Modesty.

Much of my criticism of Platonism has been mentioned already. It merely consists in saying that Platonism does not allow to meet the epistemological challenge. First, Quine-Putnam Platonism is independent from epistemological concerns. Second, Godelian Platonism does not provide a clear account of the intuitive contact between us and mathematical objects. Third, more recent attempts to give such an account rely (as far as I know) on what Maddy (1989) calls compromise Platonism and asserts that mathematical entities are in the world. But mathematical knowledge is not directly knowledge about the world, since the truth of its propositions is not assessed on the ground of what the world is. Macnamara's and Maddy's intuitions are mathematical intuitions, but strangely enough, they are not intuitions of the Platonistic mathematical realm.

Two alleged links between the worldly mathematical realm and mathematicians are put forward. First, Maddy provides an account on how mathematicians refer to sets [Maddy 1980]. Along the line of the causal theory of reference, she shows that we perceive sets and that this perception is at the origin of our intuitive knowledge about sets. Maddy forcefully describes a referential concept of set that is developed through interactions with sets of real things. This referential concept, however, remains a *naïve* concept. It is certainly the one we use in our day-to-day reasoning about sets of things and that may be at the basis of our

naïve understanding of numbers. What seems doubtful, however, is that set-theorists use the very same concept in their reasoning. In any case, a mathematician is not allowed to prove a theorem by referring to instantiated sets of real things. This, again, casts serious doubt on the referential character of mathematical concepts. The naïve concept of set does play a role in Mathematics, but it is not the one of allowing mathematicians to refer to worldly things. Mathematical notions have a referential character before (naïve theories) and after (applied mathematics) Mathematics proper, but mathematical practice allows no such reference. The second alleged link between the world and mathematical knowledge calls on evolutionary theories. It is mentioned by both Maddy and Macnamara. Innate mathematical concepts are the fruit of an evolutionary adaptation of our cognitive apparatus to the world. These concepts could not be misleading, or else they would not survive evolutionary selection. Therefore they are concepts providing true intuitive beliefs. The problem is that evolutionary selection does not guaranty truth. As Maddy acknowledges herself, intuitive beliefs may be false. Suppose, however, that mathematical intuition happened to be true intuitive beliefs, it would remain that in lack of any supplementary mean to assess the truth of a mathematical proposition, we are bound to rely on psychological facts only. The causal link does not cancel psychologism.

Thus Platonism provides no clue on the processes through which we assert the truth and the falsity of mathematical propositions.

Any purely psychological account of reasoning leads to psychologism, and psychologism downplays the normative nature of reasoning. This led some philosophers such as Davidson to urge for modesty: norms cannot be explained away in a naturalistic framework. As a consequence, Mathematical practice cannot be reduced to its causal antecedents. With this in mind, Engel (1989) attempts to provide a descriptive theory that would grasp human rationality. The research program consists of finding the laws that a person would ideally follow; it assumes that people are rational and it prescribes to abstract this rationality. Engel compares this abstraction with the abstractions made in physics (*e.g.*, a system is assumed closed) or in economics (*e.g.*, with the homo-economicus). The abstraction here is made on the empirical limitation weighting on reasoning. The idealisation is therefore not an empirical generalisation. What is taken into consideration is the intuition we have of the validity of arguments. In that way it strongly calls on common sense. What is to be built eventually is a logic that suits as well as possible our intuitions of validity. There could be two strategies for building this logic [Engel 1989, 397]. The first one is to gather our particular intuitions of validity

and from this build a system of logic that accounts for those intuitions. The second one, which seems more feasible, is to start from an already existing logic (*e.g.*, first order classical logic) and adjust it to our intuitions by making some corrections, possibly loosening the requisites for validity. An obvious adjustment, for instance, when starting with first order logic is the one required for the connector ‘if . . . then’ which is notorious for giving some counterintuitive results [Engel 1989, 44]. The idealisation can be either maximal, that is to say that we suppose a strong rationality and build a strongly normative logic, or minimal, that is to say that our requisites in the consistency are loosened in order to account for more rational actions. The first choice answers to the demand that logic be normative and guide precisely our reasoning, the second choice answers to the demand that logic be applicable and correspond to people’s actual rationality. The maximal idealisation would account only for what a completely and perfectly rational person would do. But it is not realist to ascribe such rationality to people. On the other hand, one wants to avoid the danger of just giving a description of how people generally act, and thus failing to account for rationality. Consequently, Engel advises to adopt the principle of thoughtful equilibrium between the empirical and normative constraints. This shall take into account both rationality and the restrictions set by cognitive aptitudes. The logic accordingly constructed shall give rules as close as possible to our actual reasoning. It would give an “empirical theory of deductive competence” [Engel 1989, 392]. The logical system thus discovered is justified empirically by judgements of acceptability on particular inferences, but its propositions are justified by the system, as in any formal logic. We still deal with a deductive logic and we cannot be accused of justifying inferences with psychological facts.

The first striking point is the similarity between Macnamara’s and Engel’s research programs. They both want to account for natural reasoning on the ground of a logical construction and the research programs consist in discovering the appropriate logic, which shall be in accord to our logical intuitions. The logical incompetence of actors is attributed to performance factors such as the memory space and the ambiguous interpretations of data. So the difference between the two projects may be the status which is given to the logic accordingly constructed. Macnamara asserts the existence of this logic in our mind in the form of mental devices. Engel is much more careful on this point. He must face the following dilemma: either he gives a causal status to the empirical theory of deductive competence and then he commits the ‘sin’ of psychologism; or he does not give this causal status and then we do

not see what the explanatory power of the theory is. Engel has a well-balanced discourse with regard to the psychological reality of a system of logic, once characterising it as respectable, though improbable, empirical hypothesis [Engel 1989, 413], and elsewhere stigmatising it as being an “illusion descriptive” [Engel 1989, 393]. His stance tends to be neutral with regard to the constitution of the mind. Yet, Engel assumes that, notwithstanding this neutrality, his research program is relevant to psychology. Why is that so? If the relevance is not in establishing some causal laws describing our mental processes in reasoning, where does it stem from? How does this logic contribute to the psychology of reasoning? As a matter of fact, Engel cannot attain the “thoughtful equilibrium” he is aiming at. While the content of his logic can be a compromise between ‘strongly normative’ and ‘descriptive of people’s actual behaviour’, the *status* of the resulting logic cannot. It is either the logic of the psychologist, and then one should assume that it is a description of some thought process, or the logic of the logician, and then it is purely normative. The dilemma strikes once more: it seems impossible to describe mathematical cognitive abilities without falling into psychologism.

The strategy I advocate is a way out of this dilemma: humans obviously have the ability to do mathematics, yet this ability cannot explain the normative component of mathematical knowledge. As naturalists, the obvious move for continuing our investigations is to come back to the empirical data. What is this mysterious normative component that forbids us to describe cognitive processes as the cause of mathematical knowledge? Well... normative behaviour consists in assessing each other’s work. It is first and foremost a social phenomenon.

4 The cognitive and the social

Here are a few of the social events that make up Mathematics: Mathematics is taught at school; this teaching creates a community of mathematicians; mathematicians submit their work to journals and editors, they communicate their results in the hope that their work be recognised as mathematics. The social application of norms is at work in all those events: a pupil is congratulated for his work, an article is accepted or not, a work is qualified as good or erroneous. Mathematics is made of assertions, and choosing which assertions can enter the corpus of Mathematics is a collective decision. It is decided by the community of Mathematicians. Among the infinite set of assertions, mathematicians

choose, collectively, which are true, which are nonsensical, which do not worth considering, which can gain the status of proof, definition, axiom or theorem. For instance, a theorem is actually discovered only when it has been recognised as such by the community of Mathematicians. Mathematical knowledge is not the product of solitary mental devices. Social interactions are essentially constitutive of it. As a consequence, one must rejects assertion (2) ‘Thus Mathematics is the product of mental devices’ that lead us to psychologism. Even if producing Mathematics obviously requires using our mental devices, the constitution of mathematical knowledge involve something more for constituting norms, viz. social interactions.

Psychologism is the counterpart of a simplistic understanding of mathematical production which corresponds to Figure 1.



Figure 1: The simplistic picture

Admittedly, the historical version of this picture is slightly more complex as can be seen from Figure 2.

The confusion, I believe, lies on what it means for a psychological module to produce knowledge. Knowledge is not the output of some module, but the output of some modules leads to a behavior that al-

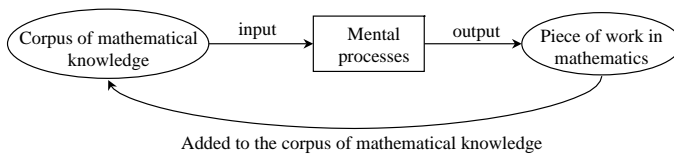


Figure 2: The incremental picture.

lows, in certain conditions, the production of a specific knowledge. For instance the behavior of good pupils in math class is made possible by their cognitive abilities. If it was impossible to teach any mathematics to anybody, then it is likely that mathematics would not exist at all. The fact that mathematics is actually learned is a psychological fact. But the teaching itself is a sociological fact. Likewise, the fact that mathematicians do produce theorems is a psychological fact. But the fact that their productions are labelled (or not) theorems, id est recognized as mathematical productions by the community, is a sociological fact. The automatic application of a technique, or the blind rule following of the individual, itself does not furnish a criterion of correct procedure, it just allows the procedure to be. This dictates a more complex picture than the above, which aims to take into account the social factors:

Just a more complex picture is shown in Figure 3.

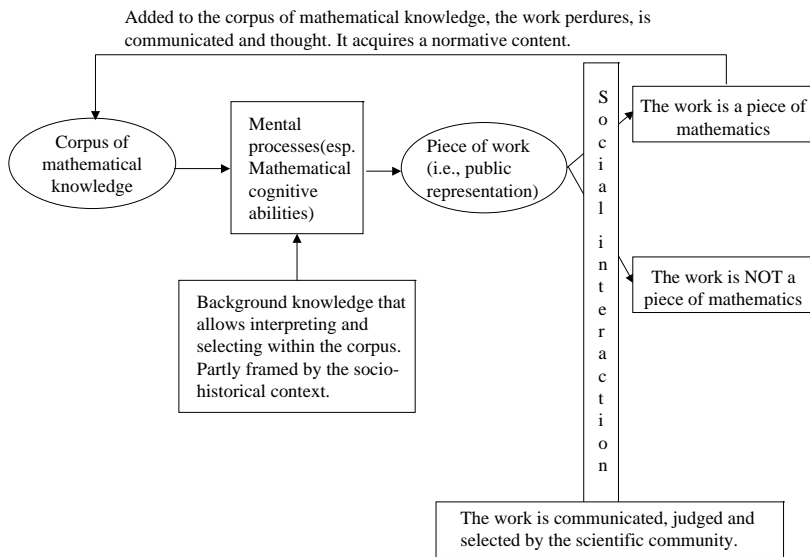


Figure 3: A more complex picture.

In this picture, the blue prints indicate the social events. It is these events that constitute the normative nature of mathematics.

Unfortunately, these social processes, although their existence is acknowledged, are not given their real importance. The opponent of the

sociology of scientific knowledge would argue that whether recognised or not by the community mathematics is done by individuals, it is the produce of their minds. Social interactions, he would go on, have no causal action on the content of Mathematics. At most, cultural components may push mathematicians in one direction or another. Thus, the more complex picture says nothing more than the simplistic ones.

I believe such a view can lead to a genuine research program. It is not, however, a question that can be settled a priori: these events are real, they are constitutive of mathematical practice, they are the manifestations of the normative nature of Mathematics. Mathematics is de facto a collective, historical production. Thus the psychological reductionist program bears the onus of proof. The point is that there cannot be a straightforward reduction of the production of mathematics to cognitive abilities (simplistic and incremental pictures). Yet, it is still open whether a more complex and fine-grained reduction is possible. A reductionist psychological program has to show that the social events of the more complex picture can be explained in terms of cognitive abilities only. Historical and social contingencies have no real causal power on the content and development of Mathematics. Such a program is noble enough, and quite ambitious. It would greatly gain, however, if it would take the challenge set by the sociology of scientific knowledge seriously. In fact, the scientific challenge, both empirical and theoretical, is huge.

Let me point out at some of the difficulties that a reductionist program would have to face:

(1) One of the fundamental assertions of sociologism is that any concept or proof procedure is under-determined by its past applications or use [Bloor, 1991, 1997]. Definitions, which are so central to mathematics, are no exception. They are nothing but single specific uses of the concepts defined that do indeed strongly constrain future applications but cannot wholly determine them. (Think of Lakatos' (1976) description of the making of mathematics and note that we can never know whether one will not appear with some kind of monster polyhedra or any other sort of problem. The way the problem will be solved will depend on the decision of the community). Hence, while Mathematics is highly constrained by cognitive abilities, it remains under-determined. Contingent, historical, social determinations come to fill the gap. The sociology of knowledge thus aims to point out the causal role of these social determinations on the content of Mathematics. A reductionist account should not simply deny these causal roles. It should on the contrary try to show that social behaviour and collective decisions are, in turn, wholly determined by our cognitive apparatus. For instance the

thesis that the “cultural creation of the real number was a platonistic re-discovery of the underlying non-verbal system of arithmetic reasoning” [Gallistel *et al.* 2002] should be confronted to historical events: Were the 18th century developments toward the concept of limit determined by our sole cognitive abilities [Heintz 2002]? What is then the significance of Non-Standard Analysis [Lakatos 1978]? It seems that even at the most basic levels some decisions are to be taken [Bloor 1994]; these decisions are taken collectively, by socially situated mathematicians.

(2) The second difficulty for the reductionist program lies, as I already pointed out, in the normative nature of mathematical practice. Only collective performance can create norms for good or bad, true or false, and only collective performance can make mathematics, since such norms are essential to it. The standards through which individual production is collectively judged is not the ideal, unconstrained, performance of cognitive abilities. Showing that social norms are wholly determined by our cognitive apparatus seems difficult. In particular, it remains to discover the processes through which semantic properties of cognitive processes are implemented, so to speak, in social norms (see *e.g.*, D. Sperber (1996) for an analysis of the relations between cultural artefacts, such as norms, and our cognitive apparatus)

5 Conclusion

There is a belief that prevents the social study of mathematics: it consists in assimilating the collective production to the individual production, the former being thought as being nothing more than the sum of the latter. This mistaken assimilation is the assumption that transforms sound cognitivism into psychologism. A proof is thought to be nothing but the correct use of our logical competence, and Logic itself is thought as the very result of our ideal performing of the logical competence. The truths of logic are based on and depends on psychological facts, they are a pure expression of mental phenomena. Yet, if mathematics is defined as the mere product of individual mental processes, then mathematics is psychological facts. These theses, of course, have been strongly criticized by Husserl and Frege, who call upon the fact that mathematical truths are objective rather than subjective, and cannot be reduced to the empirical facts studied by psychology. I have argued that the best alternative theory to psychologism is a kind cognitively informed sociologism.

There is an obvious tension between psychological and sociological theories of Mathematics. This is understandable since each discipline

is challenging the other. Sociologists point out the historical and social contingencies at work in the making of mathematics and thus render psychological reduction difficult. Conversely, Psychologists bring up empirical evidences of the cognitive determinations that make mathematicians think the way they do. This lessens the relevance and causal power of historical and social contingencies. Yet these difficulties, far from being reasons to sulk (to say the least) one another, are challenges that could be fruitfully taken.

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