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General Introduction

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General Introduction

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1 The epistemology of Otto Hölder

This special issue is devoted to the philosophical ideas developed by Otto Hölder (1859-1937), a mathematician who made important contributions to analytic functions and group theory. Hölder's substantial work on the foundations of mathematics and the general philosophical conception outlined in this work are, however, still largely unknown. Up to the present, philosophical interest in Hölder's work has been limited to his axiomatic formulation of a theory of measurable quantities published in 1901 in the article *The Axioms of Quantity in the Theory of Measurement*. This article attracted the interest, among others, of Louis Couturat, Ernst Nagel, and Patrick Suppes. More recent historical studies ([Ehrlich 2006], [Radu 2003]) have explored other aspects of Hölder's rich conception of the foundations of mathematics, against the historical background of the 19th century *Grundlagenkrise*, emphasizing the rich interdependence between Hölder's conception and ideas proposed by Immanuel Kant, Hermann v. Helmholtz, Moritz Pasch, David Hilbert, Hermann and Robert Graßmann, Giuseppe Veronese, and Rodolfo Bettazzi. These studies opened up a broad field for further historical research. A deeper understanding of Hölder's ideas requires a more detailed analysis of his logico-philosophical conception of the foundations of arithmetic, geometry, and physics. It is our conviction that this goal would benefit substantially, on the one hand, from a more widespread knowledge of Hölder's earliest methodological publications and, on the other hand, from a closer examination of the ideas found in Hölder's last and most elaborate philosophical work.

Hölder's interest for the modern reader is both historical and theoretical: from a historical point of view, Hölder belongs to a tradition of mathematicians who, like Poincaré or Weyl, investigated the epistemology of mathematics by taking the results of physics more into account than the results of logic. The general oblivion into which his epistemology fell might be related to the increased interest in the logical foundations of mathematics and set theory. Hölder's interest in other branches of mathematics, as discussed in *The Mathematical Method*, are one of the reasons for the interest in his approach: on the one hand it is oriented to case-studies and to an investigation of the practice of mathematics; on the other hand it is centered on the notion of deduction, but does not reduce deduction to the relation of logical consequence. For this reason, reading Hölder might be especially interesting for the investigation of mathematical practice and the deductive inferences that are at play in informal mathematical proofs.

2 Biography

Otto Hölder was born on December 22, 1859 in Stuttgart. He took up his studies in his native town and then moved to Berlin, where he studied mathematics with Weierstraß, Kronecker and Kummer. Hölder then moved to Tübingen, where he received his PhD in mathematics in 1882 under the direction of Paul du Bois-Reymond with a thesis in potential theory (the study of harmonic functions). In 1884 he went to Göttingen, where he obtained a PhD in philosophy and wrote his *Habilitationsschrift* on Fourier series. In 1882 he met Felix Klein in Leipzig, where he later became associate professor in 1889, after having held such a position for seven years in Tübingen and then for a further three years in Königsberg, where he became the successor of Minkowski in 1896. In Leipzig, where he received the chair of Lie, he was the advisor of students such as Emil Artin and Oskar Becker, and became member of the *Sächsische Akademie der Wissenschaften*. Hölder died on August 29, 1937.

Hölder made important contributions to analysis and group theory, but he also worked on geometry, algebra, measurement theory and epistemology. His two main mathematical results—Hölder's inequality and the Jordan-Hölder-theorem—are still named after him. An interest in philosophical questions was already present at the beginning of his career, but from 1914 on, he concentrated almost exclusively on the foundations of geometry and arithmetic and the role of deduction in mathematics and physics. The results of this work merged ten years later in a comprehensive work *Die Mathematische Methode – Logisch erkenntnistheoretische Untersuchung im Gebiete der Mathematik, Mechanik und Physik* (*The Mathematical Method. Logico-philosophical Investigations in the Domain of Mathematics, Mechanics, and Physics*), published in 1924. In this book Hölder reveals his doubts concerning the possibility of expressing all mathematics through logical symbol-

ism. From 1924 onwards, Hölder turned again to complex analysis and number theory, but continued publishing on the foundations of mathematics.

3 Content of this volume

This issue contains English translations of two early epistemological texts by Hölder—*Intuition and Reasoning in Geometry* from 1900 and his review of Robert Graßmann's *Formenlehre*, published in 1892—as well as four previously unpublished articles discussing Hölder's ideas in the light of 19th and early 20th century philosophy of mathematics.

Intuition and Reasoning in Geometry (1900) is Hölder's inaugural lecture given in Leipzig at the beginning of the academic year 1899. The published text includes extensive notes added by Hölder that constitute almost two-thirds of the volume (comparable only to Alexander von Humboldt's *Aspects of Nature*, notorious for its excessive annotation). Here, Hölder studies philosophical problems raised by the methods used in geometry and, to some degree, in mechanics. Hölder claims that no serious investigation of mathematical method—e.g., of the special character of the reasoning used in mathematical deduction from the axioms—is available. While mathematicians, on the one hand, are concerned only with the construction of various systems of axioms, and philosophers, on the other hand, are only looking for the sources of the axioms, to be found either in experience or intuition, no one investigates the deduction from axioms. In his inaugural lecture, Hölder begins a case by case investigation of deductive methods used in mathematics and mechanics. Even if a richer set of examples will be given only in his later work on *The Mathematical Method*, the inaugural lecture already contains some fundamental examples, emphasizing the use of auxiliary lines in geometric constructions and of arithmetic in the method of exhaustion. He comes to the conclusion that mathematics and the exact sciences have a logic of their own: although purely formal and completely exempt from intuition, deduction is not analytical. Rather, according to Hölder, in deduction we perform a thought experiment which replaces real experiments and no longer deals with the objects themselves, but with their mutual relations in thought.

It seems that there are not many traces of *Intuition and Reasoning in Geometry* in the secondary literature. In a review of *The Mathematical Method*, Edgar Zilsel praised the originality of the former work: "Already a quarter century ago, the author, with his inaugural lecture *Intuition and Reasoning in Geometry*, became engaged in the philosophical discussion on the foundations of mathematics—and raised its level considerably" [Zilsel 1926, 646]. Nevertheless, *Intuition and Reasoning in Geometry* made no waves in the literature and even today is known only to a small number of specialists. The only direct trace in the literature seems to be the dispute with Ernst Mach on the proof of the law of the lever (analyzed by Oliver Schlaudt in this volume). We think however that Hölder's paper rightly deserves attention today,

in particular in view of recent trends in philosophy of mathematics that focus on case studies and mathematical practice, rejecting the identification between the inference structure of mathematics and the deductive calculi developed in classical logic.

The same holds for Hölder's review of Robert Graßmann's *Theory of Number or Arithmetic*, which is—as the translator Mircea Radu rightly remarks—less a confrontation with Robert Graßmann, than a first outline of Hölder's own account of axiomatics and of its limits in the foundation of mathematics. As in the case of the inaugural lecture, this review already contains a preliminary sketch of what Hölder will later develop in the 1924 volume. Hölder compares two interpretations of Robert Graßmann's *Theory of Number*, which was meant as part of a project to restore the organic unity of the system of mathematics by examining the specific process of sign-construction involved in producing mathematical knowledge.

Biagioli's paper provides a general introduction to the philosophy of geometry at the end of the 19th century and presents the main topics developed by Hölder in the inaugural lecture. The analysis of Hölder's answers to the charge of circularity raised by the Kantians to Helmholtz's empiricism reveals that he considers geometrical concepts to be empirical inasmuch as they are derived from empirical experience, and as conditions of experience, inasmuch as they are rules that allow us to infer further facts. Yet, Hölder not only reacts to the epistemology of Kant and Helmholtz, inheriting the idea of some kind of *a priori* knowledge from the former and the empirical origin of several geometrical concepts from the second. He also discusses the classical distinction between geometry, which has axioms, and arithmetic, which is on the contrary based on definitions, and accepts the new trend in philosophy of mathematics based on the rejection of intuition in the development of rigorous proofs, further investigating the role of deduction in geometrical constructions. Biagioli's paper investigates how these features of Hölder's epistemology can be interpreted with respect to Kant's own conception and to the neo-Kantian perspective of Cassirer. On the one hand, there are some limits to Hölder's understanding of Kant's remarks on pure intuition; on the other hand, Hölder's logical analysis of mathematical reasoning as a concatenation of relations has some resemblance to Cassirer's conception of mathematics. Biagioli claims that Hölder, like Cassirer, aims to develop a methodological synthesis of Kantianism and empiricism that is grounded on a relational conception of the *a priori*.

Analyzing Hölder's reply to Mach's critique of the Archimedean proof of the Law of the Lever, Oliver Schlaudt offers an exhaustive presentation of one of the relevant case studies investigated in *Intuition and Reasoning in Geometry*. Hölder's interest in this mechanical example is related to his understanding of deduction in science: like Mach he believed that in the Archimedean proof the conclusion was implicitly contained in the premises. Yet, while Mach argued that the Archimedean proof was circular—meaning that it could not provide any further knowledge beyond the empirical facts assumed in the premises—Hölder claimed that the proof is sound because it

makes explicit something that is contained in the premises but is not epistemically accessible as such. Deduction is investigated as a means to order mathematical truths, and the difference between the premises and the conclusion is related to the nature of the relational concepts contained in them. Schlaudt associates the case study analyzed in 1899 with some remarks made by Hölder in the 1924 volume, where he develops a theory of synthesis, claiming that synthetic concepts have “hidden properties” that can be brought out in further deductive steps. The detailed analysis of quantitative and qualitative concepts and of metrical relations contained in the proof of the Law of the Lever allows Schlaudt to make a connection between the critique of Mach and Hölder’s understanding of logic, and in particular of the role of concept formation, which is a much more complicated process than nominal definitions. The paper clearly shows how a bottom-up investigation of a specific mathematical proof leads Hölder to develop an argumentative analysis of mathematical inferences that is nearer to Vailati’s analysis of mechanics than to foundational efforts to reformulate scientific theories as logical calculi.

The same idea is at the core of Hölder’s review of Robert Graßmann’s volume, as Mircea Radu shows in his contribution: disclaiming Graßmann’s idea that rigor can be reached only by means of a transformation of scientific theories into a thinking calculus—Leibniz’s utopian project—Hölder’s main aim is to criticize the widespread confidence in symbolism and limit its possible uses in mathematics. This is shown by Radu through an analysis of Hölder’s conception of axiomatics and proof: Hölder believes that logic could not itself be axiomatized, given that this would amount to the infinite regress of giving a deductive account of the science of deduction; the same holds for metamathematics, for a meta-mathematical proof of the consistency of mathematics would run into the same regress, because at some point it would require another system of axioms. Although Hölder does not support his claim with fully developed philosophical reasonings, Radu shows convincingly how he derives some arguments from the analysis of various mathematical proofs occurring in geometry and arithmetic. Relying on several examples taken from *The Mathematical Method*, Radu evaluates Hölder’s defense of the genetic against the axiomatic method in the case of arithmetic. The two methods were opposed by Hilbert in *Über den Zahlbegriff* [On the concept of number], where he remarked that even if the genetic method is mostly used in arithmetic, whereas the axiomatic method is used in geometry, the axiomatic method should be considered as preferable in both disciplines. According to the genetic method, the general notion of real number is generated by successive extensions of the concept of natural number: in particular one first defines a number—the unit—and then further generates the other numbers together with their rules. According to the axiomatic method, a set of entities is assumed as existent from the beginning, together with their relations, and one must afterwards prove the consistency of the axioms that state such relations. Radu shows that Hölder’s idea that the genetic method should be primary is related to a radically different conception of mathematical proof, based on the

synthesis of concepts and on the idea that sequences of concepts of different order are used in mathematical proofs. The genetic method is typical of mathematics and logic, which—unlike empirical sciences—are characterized by an unclear distinction between the content of the theory and its form: e.g., the sequence-concept is at the same time the content and the form of our thinking.

The difference between geometry and arithmetic is further explored in the paper by Paola Cantù, where the distinction between given and constructed concepts is analyzed in order to show that the fundamental difference between the two disciplines does not concern the kind of objects these sciences are applied to, but rather the different origin and formulation of their concepts. While geometry contains some concepts that cannot be defined synthetically, the arithmetical concepts are all synthetic—i. e., concepts that have their source in some activity, as in the genetic method described above. More precisely, arithmetical concepts are constructed in deductive processes themselves, because “the abstraction of new general concepts and rules thus forms in a way a constituent of deduction”. This analysis of the opposition “given-constructed” allows Cantù to reconstruct Hölder’s position with respect to Pasch, Helmholtz and Kant and provides several reasons why Hölder, although evidently sharing an empiricist account of geometry, did not attempt to advance a definitive argument against the Kantian understanding of mathematics. The difference between arithmetic and geometry and the role played by axioms and experience in it are then used to understand Hölder’s choice of primitive propositions in his axiomatic formulation of the theory of magnitudes, and in particular of the Archimedean axiom as a relevant property of magnitudes—even if he had developed an original non-Archimedean model. Comparing Hölder’s and Veronese’s mathematical theories, Cantù shows that the different choices made at the axiomatic level reflect different epistemological frameworks: although they both took inspiration from Pasch, Veronese was interested in the mathematical construction of a non-standard order relation that is nonetheless compatible with our intuition of physical objects, while Hölder’s was exclusively interested in the empirical foundation of measurement.

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