Philosophia Scientiae

Travaux d'histoire et de philosophie des sciences

17-1 | 2013

The Epistemological Thought of Otto Hölder

Otto Hölder's 1892 "Review of Robert Graßmann's 1891 *Theory of Number*". Introductory Note

Mircea Radu

Electronic version

URL: <http://journals.openedition.org/philosophiascientiae/817>

DOI: 10.4000/philosophiascientiae.817

ISSN: 1775-4283

Publisher

Éditions Kimé

Printed version

Date of publication: 1 March 2013

Number of pages: 53-56

ISBN: 978-2-84174-620-0

ISSN: 1281-2463

Electronic reference

Mircea Radu, « Otto Hölder's 1892 "Review of Robert Graßmann's 1891 *Theory of Number*". Introductory Note », *Philosophia Scientiae* [Online], 17-1 | 2013, Online since 01 March 2016, connection on 06 November 2020. URL : <http://journals.openedition.org/philosophiascientiae/817> ; DOI : <https://doi.org/10.4000/philosophiascientiae.817>

Tous droits réservés

Otto Hölder's 1892 "Review of Robert Graßmann's 1891 *Theory of Number*". Introductory Note

Mircea Radu

Universität Bielefeld (Germany)

Résumé : Le compte rendu de la *Théorie du nombre* de Robert Graßmann constitue la première affirmation des idées de Hölder sur la distinction entre présentation génétique et présentation axiomatique des mathématiques, qui trouvera sa formulation mature dans le livre *La Méthode mathématique* (1924). J'espère que cette traduction en anglais du compte rendu de Hölder permettra de faire connaître ce document unique à un public plus large. Dans la note introductive, je précise le contexte de l'écrit de Hölder et j'ajoute quelques commentaires concernant la traduction.

Abstract: Hölder's review of Robert Graßmann's *Theory of Number* provides the first statement of Hölder's most significant tenets concerning the distinction between the *genetic and axiomatic* presentation of mathematics, the mature expression of which is found in Hölder's book *The Mathematical Method* of 1924. By translating Hölder's review into English, I hope to make this unique document known to a wider public. In my introductory note I provide some context to Hölder's paper and a few other remarks concerning the translation work.¹

In 1891, the year in which Robert Graßmann's *Theory of Number* was published, Otto Hölder (1859-1937) was 32 whereas Robert Graßmann (1815-1901) was 76 years old. Graßmann's book is the closing chapter of a long and remarkable mathematical and philosophical tradition. This tradition goes back to the mathematical and philosophical ideas that had been gradually developed by the elder Justus Graßmann (1779-1852) beginning with 1817.² This line of

Philosophia Scientiæ, 17 (1), 2013, 53–56.

1. I thank Dr. David Marshall for native speaker advice.

2. Justus Graßmann was the father of Hermann and of Robert Graßmann. Beginning with 1817, Justus Graßmann published a series of books in which he treated several mathematical subjects and, at the same time, articulated a uniquely interesting conception about the foundations of mathematics in which philosophical as well as pedagogical ideas played a very important part. One of the most important of Justus Graßmann's works was published in 1827. Since 2009 an English translation of this work due to Lloyd Kannenberg is available [J. Graßmann 2011]. A detailed analysis of Justus Graßmann's ideas is provided in [Radu 2000].

thinking was picked up and substantially developed by Hermann Graßmann (1809-1877), Robert Graßmann's elder brother. The most important achievements of this tradition are found in the first edition of Hermann Graßmann's pioneering work *The Calculus of Extension* (*Die Ausdehnungslehre*) published in 1844 and also in the *Treatise on Arithmetic* (*Lehrbuch der Arithmetik*), which was published in 1861. The latter was planned and written together with a series of other books in close collaboration by the two brothers.³

An essential component of this tradition is the quest for restoring the *organic unity* of the *system of mathematics* and related to that, the idea that such unity can only be uncovered by examining the specific process of *sign-construction* involved in producing mathematical knowledge independently of any other disciplines. Such a treatment of mathematics must be reached without presupposing logic, language and, of course, independently of sense experience. Reorganizing the body of mathematical knowledge in accordance with this methodological ideal was the main objective of Robert Graßmann's mathematical writings. The outcome of this endeavour are found in *The Theory of Forms or Mathematics* (*Die Formenlehre oder Mathematik*) published in 1872 and in the *Theory of number* (*Die Zahlenlehre oder Arithmetik*) published in 1891.

We do not know how Hölder came to be interested in Robert Graßmann's book. It is, indeed, a bit surprising that a book mainly concerned with fairly elementary mathematical ideas attracted his attention. Hölder was a careful student of Helmholtz's work and Helmholtz praised Graßmann's 1861 treatment of arithmetic in it. This may have been Hölder's first contact with the name "Graßmann". Another link between Hölder and Graßmann may have been reached through Paul du Bois-Reymond, Hölder's Doktorvater. In his *Theory of Number*, in the effort to clarify the nature of his own approach, Graßmann provides a brief and harsh criticism of du Bois-Reymond's ideas (see below). On the other hand, from an early stage in his career on, Hölder was regarded as a scholar with a strong interest in matters of method. A project such as Robert Graßmann's with its self-declared objective of revolutionizing the presentation of mathematics may therefore have attracted Hölder's attention.⁴

Be this as it may, Hölder used the *Review* as an opportunity of presenting his own ideas concerning the foundations of mathematics. Hölder's *Review* is the expression of the clash between the genetic and the axiomatic method in dealing with the foundations of mathematics, a long time before David Hilbert's discussion of the distinction and the debate triggered by Hilbert's position were published. Hölder's discussion of the distinction provided in the *Review* contains a series of remarkable considerations about things like the relationship between existence and consistency in mathematics and about the general requirements that should be met by an axiomatic approach to the

3. Compare, for instance, [R. Graßmann 2009, 214].

4. Compare [Radu 2003, 342ff.].

foundations of pure mathematics. It also contains some important remarks about the relationship between logic, proof, and arithmetic.

Hölder basically argues that Graßmann's conception can be given both a *formal-axiomatic* as well as a *genetic* interpretation. Hölder raises a series of objections to the former and endorses the latter. In a sense, Hölder's *Review* is less a confrontation with Robert Graßmann. It is rather an outline of Hölder's own conception of axiomatics, of its proper place and limits in dealing with the foundations of mathematics. Hölder's *Review* is the first statement of an original critical philosophy of mathematics whose mature expression enfolded in a series of subsequent writings.

One difficulty in truly appreciating Hölder's ideas is caused by the style of the *Review*. In the *Review* one often finds only short fragments, hints, and allusions, where more elaborate explanations would have been needed. The other difficulty is linked to the fact that, in a sense, the *Review* requires a reader, familiar with the broad tenets of Robert Graßmann's work. In some cases I have included Hölder's original terminology in German along with the English translation. In all these cases I preserved the orthography found in the German original document, which in many cases differs from modern German orthography.

In order to render the significance of some of Hölder's statements clearer, I have included a series of footnotes in which I provide some context to Hölder's statements. The notes at the end of the document belong to Hölder.

Bibliography

GRASSMANN, HERMANN

1862 *Die Ausdehnungslehre*, Berlin: Enslin.

2000 *Extension Theory (1862)*, The American Mathematical Society, translated by Lloyd Kannenberg.

GRASSMANN, JUSTUS

2011 On the concept and extent of the pure theory of number (1827), in *Hermann Graßmann—From Past to Future: Graßmann's Work in Context. Graßmann Bicentennial Conference, September 2009*, edited by PETSCHKE, H.-J., LEWIS, C. A., LIESEN, J., & RUSS, S., Basel: Birkhäuser, 455–488, translated by Lloyd Kannenberg.

GRASSMANN, ROBERT

1872 *Die Formenlehre oder Mathematik*, Hildesheim: Georg Olms Verlagsbuchhandlung.

1891 *Die Zahlenlehre oder Arithmetik streng wissenschaftlich in strenger Formelentwicklung*, Stettin: Verlag R. Graßmann.

2009 Unpublished description of the life of Hermann Graßmann by his brother Robert (1877), in *Hermann Graßmann—Roots and Traces*,

edited by PETSCHKE, H.-J., KANNENBERG, L., KESSLER, G., & LISKOWACKA, J., Basel: Birkhäuser, 203–221.

HÖLDER, OTTO

- 1892 Review of Robert Graßmann, *Die Zahlenlehre oder Arithmetik*, streng wissenschaftlich in strenger Formelentwicklung, *Göttingische gelehrte Anzeigen*, 154(15), 585–595, Engl. tr. by M. Radu, this volume 57–70.
- 1924 *Die mathematische Methode. Logisch erkenntnistheoretische Untersuchung im Gebiete der Mathematik, Mechanik und Physik*, Berlin: Springer.

RADU, MIRCEA

- 2000 Justus Graßmann's Contributions to the Foundations of Mathematics—Mathematical and Philosophical Aspects, *Historia Mathematica*, 27, 4–35.
- 2003 A debate about the axiomatization of arithmetic: Otto Hölder against Robert Graßmann, *Historia Mathematica*, 30, 341–377.
- 2011 Axiomatics and self-reference—Reflections about Hermann Graßmann's contribution to axiomatics, in *Hermann Graßmann—From Past to Future: Graßmann's Work in Context. Graßmann Bicentennial Conference, September 2009*, edited by PETSCHKE, H.-J., LEWIS, C. A., LIESEN, J., & RUSS, S., Basel: Birkhäuser, 101–116.