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Standards of Rigor in Mathematical Practice

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## Preface

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# Preface

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From 2007 to 2011, the editors, together with Marco Panza, invited Michael Detlefsen to direct the project *Ideals of Proof* (*Chaire d'excellence senior* ANR-07-CEXC-002-01). This project focused on certain ideals, which guided mathematical reasoning throughout its history. A substantial body of publications and lectures has grown out of this project. They can be consulted under <http://halshs.archives-ouvertes.fr/IP/fr/>.

As a continuation of this project, this special issue of *Philosophia Scientiae* focuses on standards of mathematical rigor concerning the varieties of methodological, ontological, proof-theoretical questions, tackled in the framework of the philosophy of mathematical practice. Nevertheless, this volume does not only contain contributions of participants of the project, but is the result of a separate call for papers.

In the last decades, many studies (an important source is [Kitcher 1984]) have aimed to overcome classical positions in mathematical ontology and epistemology as Platonism, nominalism, formalism, but also strong anti-realism (Dummett). The failure of these efforts suggests looking at mathematical practice as a source for finding a solution to the problem these positions were willing to answer. Today, this is a crucial task for philosophy of mathematics, and for example witnessed by: C. Misak's work *New Pragmatists* [Misak 2007]; P. Mancosu's book *The Philosophy of Mathematical Practice* [Mancosu 2008]; B. van Kerkhove *et al.* *Philosophical Perspectives on Mathematical Practice* [van Kerkhove & van Bendegem 2010]; volume 16 (1) of *Philosophia Scientiae: From Practice to Results in Logic and Mathematics* [Giardino, Moktefi *et al.*

2012]; the constitution of the *Association for the Philosophy of Mathematical Practice* (2009).

The practical turn in philosophy of mathematics is by no means an uniform approach of anti-foundationalism but reflects different developments. There are pragmatic relativists who consider that “the only test for scientific concepts is whether they can be organized in a logically simple system that finds fruitful empirical applications” on arguing that “1° In doing science, we do not, in fact, have a leg to stand on. 2° We do not, in principle, need a leg to stand on” [Fine 2007, 59].

Some proponents of the practical turn pursue the heritage of Lakatos and Kuhn. Their concerns can be classified as the shift from internal to external considerations: the focus is then mathematical practice as a group or community phenomenon and as educational matters [Giardino, Moktefi *et al.* 2012, 6–7].

Finally, others shift the focus of ontological questions on topics previously considered only in passing, as evidence, visualization, understanding and explanation. Our topic in this volume, *Standards of Rigor in Mathematical Practice*, is a further issue that can be placed in this series.

Now, since the work of Poincaré it is a common place that formalized proofs do not provide mathematical understanding, that the notions of group and topology connect apparently unrelated concepts in other areas, that ontological questions are considered secondary to structural ones and that certain structures may be abandoned because of their unfruitfulness. Accordingly, it seems natural that Poincaré’s work plays a central role in this volume, all the more in a journal edited by the Poincaré Archives.

*Yacine Hamami* discusses in his paper two possible views of the validity of mathematical inference with respect to their capacity to yield a plausible account of the intuitive notion(s) of proof gap present in mathematical practice. According to the first view, a mathematical inference is valid if and only if its conclusion can be formally derived from its premises. According to the second view an inference is valid if and only if it consists in an operation that provides a ground for its conclusion given (previously obtained) grounds for its premises (Prawitz). He concludes that the ground-based account appears of particular interest for the philosophy of mathematical practice, and he finally raises several challenges facing a full development of a ground-based account of the notions of mathematical rigor, proof gap and the validity of mathematical inference.

*Ramzi Kebaïli* shows that Poincaré had implicitly in mind a personal conception of mathematical rigor that would fit with his mathematical practice. He observes Poincaré’s standard of rigor in his topological work, he studies on what grounds he is opposed to some specific standards of rigor and he develops a “Poincaréan” conception of mathematical rigor.

*Giorgio Venturi* proposes to look at set theory not only as a foundation of mathematics in a traditional sense but also as a foundation for mathematical

practice. He distinguishes between a standard set theoretical foundation and a practical one that aims to find a set theoretical surrogate to every mathematical object. After having given some examples he argues that this distinction is relevant for the philosophy of mathematics and he proposes two different kinds of foundations: a practical one and a theoretical one.

Mathematical abstraction is the process of considering operations, rules, methods and concepts divested from their reference to real world phenomena, and also deprived from the content connected to particular applications. In her contribution, *Hourya Benis Sinaceur* investigates the mathematical practice with the aim to bring to light the fundamental thinking processes at play, and to illustrate by significant examples how much intricate and multileveled may be the combination of typical mathematical techniques.

*Iris Loeb* and *Stefan Roski* argue convincingly that Bolzano supports his concept-centered methodology in the development of mathematical analysis by philosophical views, which were partially shared by working mathematicians with a formula-centered approach to analysis.

*Emylou Haffner* studies Richard Dedekind's practice of rigor in a selection of his most important works. Rigor is for him closely related to generality. The links between generality and rigor are analyzed in his theory of algebraic function, as well in his foundational essays and in his work on algebraic number theory. She analyzes the requirements for generality and the multiple conceptions of generality sustaining this demand.

*Paola Cantu* tackles the question of whether the order of concepts was still a relevant aspect of rigor in 19<sup>th</sup> and 20<sup>th</sup> century. Three case studies are taken into account: Graßmann, Peano, Gödel. The paper aims to question whether there is in fact such a stark contrast between the debate relating to the right order of concepts and the foundational question concerning modern axiomatics. The unity of the three case studies is to be found in their different but real relation to Leibniz's ideal of a universal characteristic.

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